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MECHANICS

CHAPTER I*

UNITS AND LENGTH MEASUREMENTS

MECHANICS is the science which deals with the simplest effects arisi[ng] from the application of force to matter. As such it may be regard[ed] as introductory to Physics, in that the forces and the matter o[n] which they act are taken for granted, while it is the object of Physic[s] to study more complicated cases in order to explain not only th[e] origin of the forces, but the structure of matter itself. As the simpl[e] must be dealt with before the complex, so must the principles of Mechanics be understood before the student is ready to begin the study of Physics

The branches of Mechanics with which we shall deal in these introductory pages are (1) *Kinematics*, in which motion is studied without reference to the forces which cause it. (2) *Dynamics*, with its subdivisions *Kinetics* and *Statics*. Of these, Kinetics deals with the motion of bodies, taking into account the forces and masses concerned, and Statics has to do with the conditions for the equilibrium of bodies, or their state of rest. (3) *Hydrostatics*, in which the properties of fluids at rest are the subject of inquiry.

Units.—One of the main objects of physical science, because it [is] a condition for future progress, is to obtain accurate measure[m]ents of the quantities dealt with. As a preliminary, it is necessary [to] decide in what units the results are to be expressed. The state[me]nt that a certain body "weighs 25 grams" contains two ideas: [one] is the unit—the gram; the other, the measure, states how [ma]ny times the unit is repeated—in this case 25. Each ph[ysical qua]ntity must be expressed in terms of its appropriate unit. [It is pos]sible, however, to express the more complicated units in t[erms of s]ome of the simpler ones, when it will evidently be an advan[tage to h]ave the relation between them of the simplest kind. For exan[ple, whe]n the unit of length is the foot, the simplest unit of area is [the squa]re foot, and of volume the cubic foot. A unit like the ga[llon is no]t only entirely arbitrary, but bears no simple relation to ot[her units].

units. The principle can be carried further, inasmuch as it is found that many physical quantities can be expressed in terms of thre properly chosen units. These are called the **fundamental unit** while all others bearing a more or less simple relation to them a called **derived units**. Such a system is called an absolute syste of units. The fundamental units in most scientific work are tho of length, mass and time, and the units taken are the centimet the gram and the second; hence the system is referred to as t cm.-gm.-sec. (C.G.S.) system

The centimetre is the $\frac{1}{100}$th part of a metre, the latter bei defined arbitrarily as the length of a certain platinum bar, preserv in Paris, when its temperature is that of melting ice.

The metric standard of mass is the kilogram. It is the m of a piece of platinum kept in Paris. Originally it was intended be connected with the standard of length by being defined as mass of a cubic decimetre [1] of distilled water at a temperature 4° Centigrade; now it is defined arbitrarily as above.

The gram is $\frac{1}{1000}$th of a kilogram.

The unit of time is the mean solar second, and is the $\frac{1}{86400}$th of the mean solar day. The solar day is the period between succes transits of the sun across the meridian. For various reasons interval is not constant, so an average is taken over a whole y and this is called the mean solar day.

In the British system of units, which is still used by engin the units corresponding to the centimetre, gram and second the foot, pound and second. We shall refer to this system a F.P.S. system.

Co-ordinates.—The position of a point on a plane is known perpendicular distances from two lines at right angles to each are given. Thus in Fig. 1* the lines OX, OY are called the X Y axes, O the origin, and the positions of P is known when PM or OM and ON are given. OM is called the abscissa, O ordinate of P, and the two together are referred to as the co-ordi If OM = 2 and ON = 3 units, P is referred to as the point (2, abscissa being written first. If the abscissa is drawn to th of OY it is taken as negative; similarly the ordinate is ne when drawn below OX. Thus the co-ordinates of P_1 are (− of P_2 (− 2, − 3), and of P_3 (2, − 3).

[1] A decimetre is $\frac{1}{10}$th of a metre; 1 cub. decim. = 1 litre = 1000 follows that a litre of water at 4° Cent. weighs 1000 gms. very nearly. 1·75 pints, 1 inch = 2·5400 cms.; 1 lb. = ·4536 kilograms.

the radian. If the right angle XOY of Fig. 1* is divided into 90 equal parts, each is called a degree. The degree is further divided into 60 minutes, and each minute into 60 seconds Nine degrees, six minutes, twelve seconds is written 9° 6′ 12″. The radian is defined as the angle subtended at the centre of a circle by an arc o1 length equal to the radius. Thus in Fig. 2* if arc PX = OX, the $\angle\,\theta = 1$ radian. The number of radians in $\angle$ P′OX, called its circular measure, is found by dividing the arc P′X by the radius

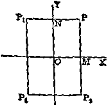

FIG. 1*.—Co-ordinates.

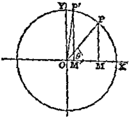

FIG 2*.—Angular Measure.

of the circle; and generally, if θ is the circular measure of a angle, θ = arc/radius.

If r is the length of the radius of a circle and C the circumference it can be proved that C/r is the same for all circles. This ratio i denoted by 2π; hence $C/r = 2\pi$, or $C = 2\pi r$.[1]

The value of π is 3·1416 or 22/7 very nearly. In Fig 2* arc XP $= C/4 = \pi r/2$,

$$\therefore \angle XOY = \frac{\text{arc XPY}}{r} = \frac{\pi r}{2}\Big/ r = \frac{\pi}{2} \text{ radians}$$

$$\therefore \frac{\pi}{2} \text{ radians} = 1 \text{ rt. } \angle$$

and $$1 \text{ radian} = \frac{2}{\pi} \text{ rt. } \angle\text{s} = 57°{\cdot}2958.$$

If an angle contains c radians and d degrees, then, expressin each as a fraction of 2 rt. $\angle$s,

$$\frac{c}{\pi} = \frac{d}{180}$$

This is a convenient equation to convert from one system ...

[1] The following ...

units. The principle can be carried further, however, as it is found that many physical quantities can be expressed in terms of three principal or main units. These are called the fundamental units, while all others bearing a more or less simple relation to them are called derived units. Such a system is called an absolute system of units. The fundamental units in most scientific work are those of length, mass and time, and the units taken are the cen . . . the gram and the second; hence this system is referred to as the cm. gm. sec. (C.G.S.) system.

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The *unit of time* is the *mean solar second*, and is the $\frac{1}{86400}$th part of the mean solar day. The solar day is the period between successive transits of the sun across the meridian. For various reasons this interval is not constant, so an average is taken over a whole year, and this is called the mean solar day.

In the British system of units, which is still used by engineers, the units corresponding to the centimetre, gram and second are the foot, pound and second. We shall refer to this system as the F.P.S. system.

Co-ordinates.—The position of a point on a plane is known if its perpendicular distances from two lines at right angles to each other are given. Thus in Fig. 1* the lines OX, OY are called the X and Y axes, O the origin, and the positions of P is known when PN and PM or OM and ON are given. OM is called the abscissa, ON the ordinate of P, and the two together are referred to as the co-ordinates. If OM = 2 and ON = 3 units, P is referred to as the point (2, 3), the abscissa being written first. If the abscissa is drawn to the left of OY it is taken as negative; similarly the ordinate is negative when drawn below OX. Thus the co-ordinates of P_1 are $(-2,$ . . . of P_2 $(-2, -3)$, and of P_3 $(2, -3)$.

[1] A decimetre is $\frac{1}{10}$th of a metre; 1 cub. decim. = 1 litre = 1000 cu. c. It follows that a litre of water at 4° Cent. weighs 1000 gms. very nearly. A litre = 1·76 pints; 1 inch = 2·5400 . . . kilograms.

Angles.—Two units of angle are in common use, the degree and the radian. If the right angle XOY of Fig. 1* is divided into 90 equal parts, each is called a degree. The degree is further divided into 60 minutes, and each minute into 60 seconds. Nine degrees, six minutes, twelve seconds is written 9° 6′ 12″. The radian is defined as the angle subtended at the centre of a circle by an arc of length equal to the radius. Thus in Fig. 2* if arc PX = OX, the $\angle\,\theta = 1$ radian. The number of radians in $\angle$ POX, called its circular measure, is found by dividing the arc PX by the radius

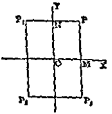

FIG. 1*.—Co-ordinates.

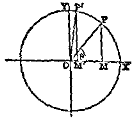

FIG. 2*.—Angular Measure.

of the circle; and generally, if θ is the circular measure of an angle, θ = arc/radius.

If r is the length of the radius of a circle and C the circumference, it can be proved that C/r is the same for all circles. This ratio is denoted by 2π; hence $C/r = 2\pi$, or $C = 2\pi r$.[1]

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$$\therefore \angle XOY = \frac{\text{arc XPY}}{r} = \frac{\pi r}{2}\Big/ r = \frac{\pi}{2} \text{ radians}$$

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and $$1 \text{ radian} = \frac{2}{\pi} \text{ rt. } \angle\text{s} = 57°\cdot2958.$$

If an angle contains e radians and d degrees, then, expressing each as a fraction of 2 rt. $\angle$s,

$$\frac{e}{\pi} = \frac{d}{180}$$

This is a convenient equation to convert from one system to the other.

[1] The following results should be remembered: area of a circle $= \pi r^2$; surface of a sphere $= 4\pi r^2$; volume of a sphere $= \frac{4}{3}\pi r^3$.

units. The principle can be carried further, inasmuch as it is
that many physical quantities can be expressed in terms of
properly chosen units. These are called the fundamental
which all others bearing a more or less simple relation to the
called derived units. Such a system is called an absolute s
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In Fig. 3* (b) let $\angle$ P'OM' $=\theta$; make $\angle$ POM $= \theta$, OP' $=$ O] d draw PM, P'M' $\perp$ OX, Then P'M' $=$ PM,

d $$\angle \text{POM}' = 2 \text{ rt. } \angle\text{s} - \theta = \pi - \theta,$$

;o $$\sin(\pi - \theta) = \frac{\text{PM}}{\text{OP}} = \frac{\text{P'M'}}{\text{OP'}} = \sin\theta,$$

$$\cos(\pi - \theta) = \frac{\text{OM}}{\text{OP}} = \frac{-\text{OM}'}{\text{OP}'} = -\cos\theta$$

is OM is taken positive, OM', drawn in the opposite direction, egative.) For example, $\sin 120° = \sin(180° - 60°) = \sin 60°$.

In Fig 2* let θ be a very small angle; then both P and M a ear X, and are PX $=$ semi-chord PM very nearly. Hence for sma alues of θ we may put $\theta = \text{PM/OP}$ which is frequently useful. ke circumstances $\sin\theta = \text{PM/OP} = \theta$, and $\tan\theta = \text{PM/OM} = \text{PM/O}$ nearly) $= \theta$.

When P coincides with X, $\theta = 0$; also PM $= 0$, and M coincid vith X. Hence

$$\sin 0° = \text{PM/OP} = 0/\text{OP} = 0,$$
$$\cos 0° = \text{OM/OP} = \text{OX/OP} = 1.$$

Similarly (Fig. 2*) when $\angle$ P'OX $= 90°$, P' coincides with P'M' $=$ OP' and OM' $= 0$.

$$\therefore \sin 90° = \frac{\text{P'M'}}{\text{OP'}} = \frac{\text{OP'}}{\text{OP'}} = 1,$$

and $$\cos 90° = \frac{\text{OM}'}{\text{OP}'} = 0.$$

In Fig. 4* (a) let $\angle$C $= 45°$, then $\angle$A is 45° if B is a right ang and BC $=$ AB. Also $\text{AC}^2 = \text{AB}^2 + \text{BC}^2$, hence if AB $=$ BC $=$

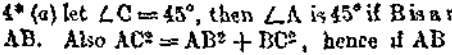


Fig. 4*

AC $= \sqrt{2}$, and from the $\triangle$ABC the trigonometrical ... can be found at once ...

EXAMPLE.—Find the number of radians in 120°.

$\frac{c}{\pi} = \frac{120}{180}$, whence, putting in the value of π, $c = 2.09$ radians

Trigonometrical ratios.—A few results from trigonometry are inserted here for future reference. Let a revolving line OP start from OX (Fig. 3*) and sweep out the $\angle$ XOP. Draw PM $\perp$ OX produced if necessary as in (b). Let $\angle XOP = \theta$; OP is the hypotenuse of the $\triangle$ POM. Then we have the following definitions:

$$\text{sine } \theta = \frac{PM}{OP} = \frac{\text{side opposite the angle}}{\text{hyp.}}, \text{ (written } \sin\theta\text{)},$$

$$\text{cosine } \theta = \frac{OM}{OP} = \frac{\text{side adjacent to angle}}{\text{hyp}}, \text{ (}\quad ,, \quad \cos\theta\text{)},$$

$$\text{tangent } \theta = \frac{PM}{OM} = \frac{\text{side opposite to angle}}{\text{side adjacent to angle}}, \text{ (}\quad ,, \quad \tan\theta\text{)}.$$

In Fig. 3* (b) if OM' is taken positive OM must be negative, and

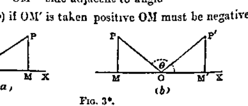


FIG. 3*.

vice versa The values of these ratios for all angles up to 90° have been tabulated.

Note that $\quad \frac{\sin\theta}{\cos\theta} = \frac{PM}{OP} \div \frac{OM}{OP} = \frac{PM}{OM} = \tan\theta.$

Hence when $\sin\theta$ and $\cos\theta$ are known, $\tan\theta$ can be calculated.

Also $\sin^2\theta + \cos^2\theta = \frac{PM^2}{OP^2} + \frac{OM^2}{OP^2} = \frac{PM^2 + OM^2}{OP^2} = 1.$

(Euclid I. 47.)

The three interior angles of a triangle = 2 rt. $\angle$s.

$$\therefore \text{(Fig. 3* (a))} \angle\theta + \angle OPM = 1 \text{ rt. } \angle = \frac{\pi}{2}$$

and

$$\angle OPM = \frac{\pi}{2} - \theta,$$

$$\therefore \sin\left(\frac{\pi}{2} - \theta\right) = \sin OPM = OM/OP = \cos\theta.$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta.$$

In Fig. 3* (b) let $\angle$ P'OM' $=\theta$; make $\angle$ POM $=\theta$, OP' = O and draw PM, P'M' $\perp$ OX. Then P'M' = PM,

and $$\angle \text{POM}' = 2\text{ rt. } \angle\text{s} - \theta = \pi - \theta,$$

also $$\sin(\pi-\theta) = \frac{\text{PM}}{\text{OP}} = \frac{\text{P}'\text{M}'}{\text{OP}'} = \sin\theta,$$

$$\cos(\pi-\theta) = \frac{\text{OM}}{\text{OP}} = \frac{-\text{OM}'}{\text{OP}'} = -\cos\theta$$

(As OM is taken positive, OM', drawn in the opposite direction, i negative.) For example, $\sin 120° = \sin(180° - 60°) = \sin 60°$.

In Fig 2* let θ be a very small angle; then both P and M ar near X, and arc PX = semi-chord PM very nearly. Hence for smal values of θ we may put θ = PM/OP which is frequently useful. In like circumstances $\sin\theta$ = PM/OP $=\theta$, and $\tan\theta$ = PM/OM = PM/OX (nearly) $=\theta$.

When P coincides with X, $\theta = 0$; also PM = 0, and M coincides with X. Hence

$$\sin 0° = \text{PM}/\text{OP} = 0/\text{OP} = 0,$$
$$\cos 0° = \text{OM}/\text{OP} = \text{OX}/\text{OP} = 1.$$

Similarly (Fig. 2*) when $\angle$ P'OX = 90°, P' coincides with Y, P'M' = OP' and OM' = 0.

$$\therefore \sin 90° = \frac{\text{P}'\text{M}'}{\text{OP}'} = \frac{\text{OP}'}{\text{OP}'} = 1,$$

and $$\cos 90° = \frac{\text{OM}'}{\text{OP}'} = 0.$$

In Fig 4* (a) let $\angle$C = 45°, then $\angle$A is 45° if B is a right angl and BC = AB. Also AC² = AB² + BC²; hence if AB = BC =

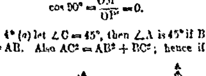


Fig 4*

AC $=\sqrt{2}$, and from the $\triangle$ABC the trigonometrical ratios for 4 can be found at once. In (b) let ABC be an equilateral triang

nd AD be $\perp$ BC. Then $\angle C = 60°$, and $\angle CAD = 30°$. Also if D = 1, AC = 2; and, as $AC^2 = CD^2 + AD^2$, $AD = \sqrt{3}$. Hence ll the sides of $\triangle$ ACD are known, and therefore the trigono- metrical ratios for 30° and 60°. The student should calculate the atios for 120°, 135° and 150° from the formulæ sin $(\pi - \theta) =$ in $(180° - \theta) = \sin\theta$, &c., by putting $\theta = 60°$, 45° and 30° in uccession. The results for $\sin\theta$ and $\cos\theta$ are here tabulated.

	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

The Vernier.—It scarcely ever happens in practice that a length o be measured is equal to an exact number of divisions on the measuring scale. The vernier is a device which enables a fraction f a division to be estimated with accuracy, and it applies equally well to circular scales. Fig. 5* represents a rough model of a calliper gauge; Q is the scale, and the part V attached to the movable jaw B is the vernier. The zeros of the two scales coincide when the jaws A, B are in contact, so that the position of the vernier zero gives the length f the body P between the jaws. In the Fig 10 vernier divisions re equal to 9 scale divisions, hence 1 v. div. $= \frac{9}{10}$ scale div. n l the difference between a vernier and scale division is $\frac{1}{10}$th f a scale division. It is seen that 7 on the vernier coincides with scale division; hence 6 on the vernier is $\frac{1}{10}$th division to the ght of its corresponding scale division, number 5 is $\frac{2}{10}$th to the ght of the next scale division, and so on, until we reach the vernier ero, which is found to be ·7 of a scale division to the right of number on the scale. The length of P is, therefore, 7·7 scale divisions. If the vernier had coincided with a scale division the required would have been 7·8. And generally, if n divisions on the re equal to $(n-1)$ on the scale, it will be seen, by similar

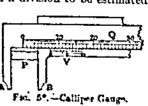


Fig. 5*.—Calliper Gauge.

scale division. This is the difference between two divisio... fraction to which it enables us to read. This is the "least count" of the vernier, and is th...

The Micrometer Screw.—An accurate screw is frequently ...ed in one form or other for length measurements. As an example, suppose the pitch of the screw is 1 mm, *i.e.* for each revolution the screw advances 1 mm. Let also the screw have a large circular ... divided into 100 equal parts. Then it is evident that the ... can be turned through $\frac{1}{100}$th of a revolution, and its point ad... or drawn back by 0·01 mm. Fig. 6* shows the application ... principle to the screw gauge. The screw Q is advanced throu... nut N by turning the milled head H, which is divided into 50 ... parts by the scale S. Suppose the pitch is ·5 mm. A $\frac{1}{2}$ mm ... R is engraved on the nut, and when the jaws P and Q are in c...

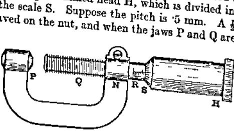


FIG. 6*.—Micrometer Screw Gauge

the zeros of S and R should coincide. For each complete tur... the screw the bevelled edge at S moves over one division of scale ... fractions of a turn are given by the circular scale S. One divi... on S corresponds to a movement on the part of Q of $\frac{1}{50}$ of $\frac{1}{2}$ a m... *i.e.* to ·01 mm. Hence the diameter of a piece of wire in cont... with the jaws P and Q is given to the nearest ·5 mm. by the scale ... while the scale S gives the amount to be added to this, in $\frac{1}{100}$ths ... mm, to get the diameter accurately.

In the spherometer (Fig. 7*) T is the circular head, divided i... 100 divisions, B is the screw of known pitch, say ·5 mm., and t... mm. scale A gives the number of complete revolutions, as in t... preceding case. One division of T, therefore, corresponds to $\frac{1}{100}$ ... mm $= \frac{1}{200}$ mm. $=$ ·005 mm. The points of the legs P, Q, R are ... one plane and the lines joining them form an equilateral triang... Fig. 8* (*b*)). The spherometer is placed on a flat sheet of glass an... the screw is turned until S is a little lower than P, Q, R; if it b... pushed at A the whole instrument then revolves about S. The centr...

EXAMPLES ON CHAPTER I*

1. Find the number of radians in 30°, 50°, 130°.

2. Calculate the tangents of 120°, 150°, 30°.

3. A barometer scale is graduated in mms, and 20 vernier divisio[illegible] 19 scale divisions. To what fraction of a mm can it be read?

4. The circular scale of a spectrometer is graduated in half degrees, [illegible] vernier divisions correspond to 29 scale divisions. What is the least [illegible] distance that can be read on the instrument?

5. A microscope scale is divided into half mms; what kind of verni[illegible] enable one to read to ·01 mm?

6. The circular scale divisions of a polarimeter are each equal to one q[illegible] of a degree, and 25 divisions on the vernier are equal to 24 on the scale. [illegible] fraction of a degree can be read on this instrument?

y. It may be positive or negative; in the latter case it is imes called a retardation. When the velocity changes by the amount every second the acceleration is said to be constant, t is measured by the increase in velocity per second. A body nit acceleration, in the C.G.S. system, when its velocity increases L per sec. every sec. Students should note that a velocity is sured in cms. per sec., but an acceleration in cms. per sec. per sec.; is often written cms /sec.^{2}.

Kinematical equations.—If a body moves for t secs. with a orm velocity u cms./sec., the distance it travels $s = ut$. Let a y move in a straight line, starting with velocity u; let its constant leration be a, the velocity after time t be v, and the distance n the starting point after time t be s cms. There are three portant relations between these quantities which will now be ived.

The increase in velocity each second $= a$

$\therefore$ the increase in t secs. $= at$

$$\therefore \quad v = u + at \qquad (1)$$

the increase in velocity is uniform the average velocity $=$ $+ v)/2$.

CHAPTER II*

KINEMATICS AND KINETICS

· to simplify matters as much as possible it will often be d that the bodies dealt with in this chapter are concentrated h small volumes that any effects arising from their dimensions neglected. Such concentrated masses are called material · es. A rigid body is one whose particles remain at fixed distnces from each other when it is acted upon by forces. Actually o body is perfectly rigid.

Velocity.—When a particle undergoes a displacement from one sition to another, both the distance it has moved and the direction its displacement must be given in order to define completely its w position. Such quantities which involve direction as well as agnitude are called vector quantities. Those which have magnide only are called scalars. **The velocity of a body is its rate of displacement**; it is therefore a vector quantity, since it involves direction. A particle has a uniform, or constant, velocity when it moves over equal distances in equal intervals of time, however small these intervals are taken. If an aeroplane has a constant velocity of 90 miles/hour it must travel 132 ft. every second, ·132 ft. every thousandth of a second, and so on. A particle is said to have unit velocity when it passes over unit distance in unit time. In C.G.S. units this is 1 cm./sec. When the velocity of a body is variable, it is still possible to define its velocity at any point. Take a short length s of its path including the point, and let t be the time taken to traverse it. Then the average velocity over this ' ·it The velocity at the point is defined as the valu T therefore t, are made very small the average velocity over a small any change of velocity in the

Acceleration.—When said to be accelerate

It may be positive or negative; in the latter case it is s called a retardation. When the velocity changes by the ount every second the acceleration is said to be constant, measured by the increase in velocity per second. A body cceleration, in the C.G.S. system, when its velocity increases r sec. every sec. Students should note that a velocity is in cms. per sec., but an acceleration in cms. per sec per sec.; ten written cms./sec.^{2}.

natical equations.—If a body moves for t secs. with a velocity u cms./sec., the distance it travels $s = ut$. Let a ve in a straight line, starting with velocity u; let its constant ion be a, the velocity after time t be v, and the distance s starting point after time t be s cms. There are three it relations between these quantities which will now be

increase in velocity each second $= a$

$\therefore$ the increase in t secs. $= at$

$$\therefore \quad v = u + at \quad \ldots \ldots \ldots \quad (1)$$

increase in velocity is uniform the average velocity $=$ 2.

$$\therefore \text{ space passed over, } s = \frac{(u + v)t}{2}.$$

ite the value of v from (1) and

$$s = \frac{(2u + at)\,t}{2}$$

$$\text{i.e. } \quad s = ut + \tfrac{1}{2}at^2 \quad \ldots \ldots \ldots \quad (2)$$

se equations give v and s in terms of t; if the velocity after over a distance s is required the quantity t must be elimi- Square both sides, of (1), then

$$\begin{aligned} v^2 &= u^2 + 2uat + a^2t^2 \\ &= u^2 + 2a(ut + \tfrac{1}{2}at^2). \end{aligned}$$

antity in brackets is s,

$^2 + ^2$

EXAMPLES ON CHAPTER II.

...er moving over 625 ft. from rest, a body has a velocity of 125 ft. p... celeration. (L. '84.)

...t of water is projected against a wall so as to strike it at right an... city of the jet be 80 ft./sec. and 100 lbs. of water strike the wall p... at pressure will be exerted against the wall, (1) when the water s... ll, (2) when it rebounds with a velocity of 10 ft./sec.? (L. '8...

...weighing 25 gms., and moving with a velocity of 300 metres... npact against a bone, being brought to rest in a distance of 1... ng. Calculate the average force exerted by the ball...

...ighing 240 lbs. is lowered with uniform acceleration down... pit, the velocity changing from 100 to 200 ft. per sec. wh... 600 ft. Determine the force exerted by the cage on the ... l. (L. '84.)

...gm. hangs over the edge of a smooth horizontal table, and... d to a mass of 980 gms., which slides without friction on... energy, in gms.-cms., lost by the system in 10 sec... ($g=981$.) (L. '93.)

... 20 gms. ...

7. What is the horse power of an engine which can pump 1000 gallons of water per minute from a well and project it with a velocity of 80 ft./sec. through a nozzle which is at a height of 40 ft. above the surface of the water in the well? [A gallon of water weighs 10 lbs.] (L. '82.)

8. A reservoir of water of area 330,000 sq. ft. is initially at a depth of 10 ft. How many ft.-lbs. can it supply to a turbine on a level with the bottom, and what horse power can it maintain on the average if it is emptied in 10 hours? [1 cu. ft. of water weighs 62½ lbs.] (L. '94.)

Let P be parallel to the base as in Fig 19*. P to be taken ... to a force, so that if it were turned through 90°, as in the ...

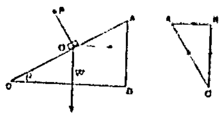


Fig 19* Inclined Plane

fig. to the right, each would be parallel to ... in the same direction as ... force. Hence △ABC ... be taken as the triangle of forces, the side perpendicular to a force being taken to represent the force. (This property should be remembered.)

$$\therefore \quad P/W = h/b = \tan\theta,$$

and

$$R/W = l/b = 1/\cos\theta.$$

In either case, if a force slightly greater than P were applied, the weight would move up the plane. The formulæ show the advantage of such an arrangement in raising a weight, as, by proper inclination, P can be made much less than W. It is much easier to push a barrel up an inclined plank than to raise it vertically, but the work done against gravity is the same in each case, viz. *mgh* ergs, where *m* is the mass raised in gms (p. 20*). This suggests a still simpler method of obtaining the results; for the gain in potential energy must be equal to the work done. Suppose the mass is dragged from C to A: the gain in energy, in gravitational units, is W*h*; the work done by R is zero, since the displacement is perpendicular to it; and in Fig. 18* the work done by P = P*l*.

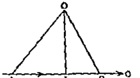
$$\therefore \quad Pl = Wh \quad \text{or} \quad P = W \,.\, h/l \text{ as before.}$$

In the second case the work done by P = P*b*, because *b* is the displacement parallel to P's direction.

$$\therefore \quad Pb = Wh \quad \text{or} \quad P = W \,.\, h/b.$$

Moment of a Force.—When it is a question of the equilibrium of a particl... tion can be confined to the possibility of motions of transl... with rigid *bodies*, whose dimensions affect the result, it i... take into account rotations also. Let AC (Fig 20*) ...on of a force whose magnitude is P, and OL the ...n O to AC; then P . OL is called the torque or the ...e round O. To see its physical significance let ... hung from a lever supported at O (Fig. 21*), and P be moved until a balance is obtained. Also

amount and get a balance as before. It will be found in every that W . AO = P . OB, *i e* the moments of the forces P and W d O are equal and tend to turn the lever in opposite directions the lever balances when the turning effects neutralise each r; the moment of P round O is therefore a measure of its turning t. Moments are taken of opposite sign according as they tend urn a body in a clockwise or anti-clockwise direction. If a force



ig. 20*.—Moment of a Force.

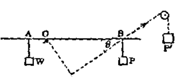


Fig 21*.—Illustration of a Moment.

ot perpendicular to the lever, as P' in Fig 21*, it may be resolved o its components along and perpendicular to OB: the former ses through O and has no moment round that point; the latter qual to P' . sin θ, and its moment round O is P' sin θ . OB. Instead this we might have drawn OL $\perp$ P', when the moment is P' . OL, t as OL = OB sin θ, the result is the same as before. In Fig. * let AB represent P in size; then the moment P . OL = AB . OL twice area of $\triangle$OAB, showing that a moment can be represented aphically by an area.

Moment of the Resultant of Two Forces.—Let OA, OB, be the es of action of two forces and that of their resultant (Fig. *). We will prove that the m of the moments of the two rces round any point M is ual to the moment of their sultant round the same point. raw FME ∥ OB and EL ∥ OA he three forces are now repre- nted by OF, OL and OE, and

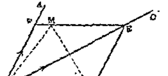


Fig. 22*.—Resultant Moment.

eir moments by 2$\triangle$OFM, 2$\triangle$OLM, and 2$\triangle$OEM The last two oments are opposite in sign to the first. We have to prove that

$$2\triangle OLM - 2\triangle OFM = 2OEM.$$

As $\triangle OLM$ is on the same base and between the same parall $\square$ FL,

$$\therefore 2\triangle OLM = \square FL = 2\triangle OFE.$$
$$\therefore 2\triangle OLM - 2\triangle OFM = 2\triangle OFE - 2\triangle OFM$$
$$= 2\triangle OEM,$$

so proving the proposition. Being true for two forces, it is true their resultant and a third force, and so on for any number of It follows from this proposition that the sum of the moments of number of forces about any point in the line of action of their resul is zero.

Parallel Forces.—In the last fig. let O be supposed to move to a very great distance along OC, then OF and OL become par and their resultant OE becomes e to their sum or difference, accor as they are in the same or oppo directions. In the first case they called like and in the second un parallel forces. The position of resultant is found by the theorem the last paragraph. Let two par forces P and Q act at A and B (23*), and let their resultant R act C. The forces are like in (a) and like in (b). In (a) $R = P + Q$, and in (b) $R = Q - P$ (if $Q > P$

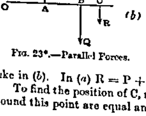


FIG. 23*.—Parallel Forces.

To find the position of C, remember that the moments of P an round this point are equal and opposite,

$$\therefore P \cdot AC = Q \cdot BC \quad \text{or} \quad AC/BC = Q/P$$

in both cases. Evidently in (b) the point C must fall outside AB the forces are to have oppositely directed moments, and the re just obtained shows it is nearer the larger force. It is often m convenient to take moments round some other point O, and express that the sum of the moments of P and Q is equal to t of R.

$$R \cdot OC = (P + Q)OC = P \cdot OA + Q \cdot OB,$$
$$(Q - P)OC = Q \cdot OB - P \cdot OA,$$

the position of C is found. Next suppose the direc- rces are changed so that they all make an angle θ with their moment round O each force is resolved perpend

is multiplied by its distance
θ, etc., and the last equation

$$\theta - P \cdot OA \cdot \sin \theta.$$

result is the same as before,
C is called the centre of the
forces acting at fixed points
acts however their direction

ple is given of a number
Another system is of special
d as being built up of in-
each being equivalent to
e resultant of these is the
acts—the centre of parallel
e body. If it be supported
rium in all positions. For
dy can be supposed to be

like parallel forces is called
is zero
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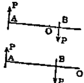


FIG. 24*.—A Couple.

nent of the couple, and
work will be done. To
to make one complete
angle turned through
ribes a circle of radius
whose circumference is

$\cdot 2\pi$

described by arm

As $\triangle OLM$ is on the same base and between the same parallel $\square$ FL,

$$\therefore 2\triangle OLM = \square FL = 2\triangle OFE.$$
$$\therefore 2\triangle OLM - 2\triangle OFM = 2\triangle OFE - 2\triangle OFM$$
$$= 2\triangle OEM,$$

so proving the proposition. Being true for two forces, it is true their resultant and a third force, and so on for any number of for It follows from this proposition that the sum of the moments of number of forces about any point in the line of action of their resul is zero.

Parallel Forces.—In the last fig let O be supposed to move to a very great distance along OC, then OF and OL become para and their resultant OE becomes eq to their sum or difference, accord as they are in the same or oppo directions. In the first case they called like and in the second un parallel forces. The position of resultant is found by the theorem the last paragraph. Let two para forces P and Q act at A and B (F 23*), and let their resultant R act C. The forces are like in (a) and like in (b). In (a) $R = P + Q$, and in (b) $R = Q - P$ (if $Q > P$

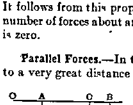


FIG. 23*.—Parallel Forces.

To find the position of C, remember that the moments of P and round this point are equal and opposite,

$$\therefore\ P.AC = Q.BC \quad\text{or}\quad AC/BC = Q/P$$

in both cases. Evidently in (b) the point C must fall outside AB the forces are to have oppositely directed moments, and the re just obtained shows it is nearer the larger force. It is often m convenient to take moments round some other point O, and express that the sum of the moments of P and Q is equal to moment of R.

Then in (a) $\quad R.OC = (P + Q)OC = P.OA + Q.OB,$
and in (b) $\quad (Q - P)OC = Q.OB - P.OA,$

and again the position of C is found. Next suppose the direc of the forces are changed so that they all make an angle θ with To get their moment round O each force is resolved perpendic

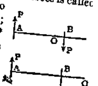
...from O. ... component is multiplied by its distance ...becomes. The components are P sin θ, etc, and the last equation

$$(Q - P)OC \cdot \sin\theta = Q \cdot OB \quad \sin\theta - P \quad OA \quad \sin\theta.$$

Dividing out by sin θ it is seen that the result is the same as before, ...nd the position of C is unchanged. C is called **the centre of the ...parallel forces.** The centre of parallel forces acting at fixed points ...s the point at which their resultant acts however their direction ...es changed.

Centre of Gravity.—On p 318 an example is given of a number ...f parallel forces acting on a magnet. Another system is of special ...mportance. Any body can be regarded as being built up of in...numerable small particles, the weight of each being equivalent to ... force acting vertically downwards. The resultant of these is the ...weight of the body, and the point where it acts—the centre of parallel ...orces—is called the **centre of gravity** of the body. If it be supported ...t this point the body will be in equilibrium in all positions. For ...many purposes the whole mass of a body can be supposed to be ...oncentrated at its centre of gravity.

Couples.—A system of two equal but unlike parallel forces is called ...couple. The resultant of such a system is zero ...d it can produce no motion of translation; ...ere may, however, be rotation. In Fig. 24* ...aw AB perpendicular to the direction of the ...o forces P, and take moments about any point ...in this line. The resultant moment is



(a) $P \cdot AO + P \cdot OB = P \cdot AB,$

...d in (b) $P \cdot AO - P \cdot OB = P \quad AB.$

FIG. 24*.—A Couple.

This constant moment is called the moment of the couple, and ... is its arm. If a couple produces rotation work will be done. To ... its amount suppose AB (Fig. 24* (a)) to make one complete ...olution round O, P remaining ⊥ AB. The angle turned through ...rt. ∠s = 2π radians (p. 3*); also B describes a circle of radius ...and circumference 2π . OB, and A one whose circumference is ...OA.

...k done = P(2π . OA + 2π . OB) = P . AB . 2π

Work = (mom. of couple) × angle in radians described by arm.

...will be in ergs if P is in dynes and AB in cms.

Conditions of Equilibrium. For a system of [illegible] [illegible] as [illegible] are considered here, it being assumed the forces are all in one plane.

(1) For a Particle. If there are two forces they must be equal and opposite. If there are three forces they must act in parallel and their resultant [illegible], or if not parallel, they must be capable of being represented by the sides of a triangle taken in order. No matter how many forces there are, if there be equilibrium the components in two directions at right angles must vanish.

(2) For a Rigid Body. (Both translation and rotation to be taken into account.) For no translation the resultant must be zero; for no rotation the sum of the moments round any point must be zero. If all the forces pass through one point they cannot produce rotation, and the conditions are the same as for a particle. If they do not all pass through the same point, the simplest conditions are that the sum of the components in two directions at right angles should vanish, and the moments round any point should vanish.

Machines. A machine is a contrivance for overcoming a resistance at one point by the application of a force, usually at another point. The resistance W to be overcome is called the weight or the load, and the applied force P the power (not to be confused with power defined on p. 19*). If P is the force that just balances W, the ratio W/P is called the mechanical advantage of the machine. The inclined plane has already been dealt with.

Levers are divided into three classes according to the position of the point about which they turn—called the fulcrum. The three types are indicated in Fig. 25*, where C is the fulcrum, P the power, and W the load. In addition to P and W the only other force acting on the lever is the reaction R at the fulcrum. Hence the resultant of P and W must act at C and be equal and opposite to R (no translation). Also the sum of the moments of P and W about any point on the line of action of their resultant is zero, that is, in all three cases

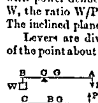


Fig. 25*.—Levers.

$$P \cdot AC = W \cdot BC,$$

R is the algebraic sum of P and W. Examples of the [illegible] first class are a pump handle, the lever used by tramway [illegible] move a [illegible] [illegible]ad points, the pole and cradle used to [illegible]

sheep into the tub holding sheep dip, and the common balance
Double levers are a pair of scissors or pincers. Examples of the
second class are a pair of nut-crackers and a wheelbarrow. In the
third class W<P, and there is always mechanical disadvantage
but in many cases space is saved. The forearm is an example;
fulcrum is the elbow joint, the power is applied obliquely by
biceps muscle at a point below the elbow, and the weight is held
the hand. Double levers are the forceps in a box of weights a
sugar tongs. If the weight of the lever, W_1, has to be taken in
account, there is an additional force W_1 acting at the centre
gravity G of the lever, but the sum of the moments round C is at
zero.

The wheel and axle is shown in Fig. 26*. Examples are the
capstan used in ships to raise
the anchor, and the windlass
used to raise water from a well.
The wheel is represented by
the radius of the handle.)
Suppose P just balances W
let radius of axle $= b$, and
radius of wheel $= a$. Then in

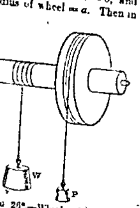


Fig. 26*.—Wheel and Axle.

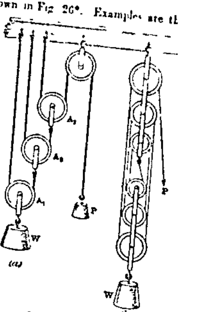


Fig. 27*.—Pulleys.

complete revolution when W is raised, the work done by P =
done on W (p. 28*), *i.e.* $P \cdot 2\pi a = W \cdot 2\pi b$, or $Pa = Wb$
of the various systems of pulleys we will deal with two only,
for the sake of illustration two methods of calculating the

D*

mechanical advantage will be used. In the Archimedes system (Fig. 27* (a)) a separate string passes round each pulley. Let W ascend x cms; then A_2 ascends $2x$ cms.; A_3 goes $2^2 . x$ cms., and P ascends $2^3 \; x$ cms. Also work by P = work done on W,

$$\therefore \; P . 2^3 \; x = W . x \quad \text{or} \quad W/P = 2^3.$$

In the common system (Fig. 27* (b)), the pulleys of the top block are usually all mounted on the same axis, and similarly with those of the bottom block. The same string passes round all the pulleys and, since it supports the power, its tension is P. As there are strings going to the lower block the upward pull is 6P. If there are n pulleys $W = n . P$. The weights of the pulleys have been neglected in each case.

The Balance.—In principle the common balance is simply a lever of the first class with equal arms. The beam AB (Fig. 28*) turns round a fulcrum C, which, to diminish friction, is made of an agate or steel knife-edge resting on a smooth agate plane. Let W be the weight of the beam and pointer, G be their centre of gravity, and suppose that unequal weights P and Q hang from the arms of length a and $CG = b$. Suppose the beam tilted through an $\angle \theta$. Taking moments round C,

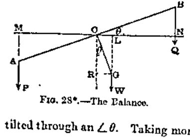


FIG. 28*.—The Balance.

$$P . CM = W . CL + Q . CN.$$

Also $\quad CM = CN = a \cos \theta$ and $CL = GR = b \; \sin \theta.$

$$\therefore \; Pa \cos \theta = Wb \sin \theta + Qa \cos \theta \quad . \quad . \quad . \quad (1)$$

$$\therefore \; \frac{\sin \theta}{\cos \theta} = \tan \theta = \frac{(P - Q)a}{W . b} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

A good balance should have the following characteristics: (1) It should be true; *i.e.* the beam must be horizontal when equal weights, or no weights, are in the pans. This is secured by making the arms exactly equal in length and weight, and the pans equal in weight. (2) It must be sensitive. This means that for a small difference between P and Q the angle of tilt, θ, must be large. For a given difference between P and Q the equation shows that tan θ, and therefore θ, is large when a is large, and W and b are small. The beam must be long and light, and its centre of gravity near

) It must be stable, *i.e.* it must return quickly to its position
st when deflected, with equal weights in the pans. Equation
iows that when P and Q are equal the only restoring couple ari
om the weight of the beam. Hence for stability Wb must
rge. (Of course the C.G. must be below C.) It can also be sho
iat for a quick swing a light beam is required. Evidently
onditions for (2) and (3) are at variance and in practice a comprom
must be effected. The scientist requires sensitivity even at
acrifice of some speed in weighing, while for a grocer a less accur
ombined with speed is sufficient.

Suspended Bodies.—When a body is suspended so that it
urn freely round its point of suspension O (Fig. 29*), it
vill come to rest with its centre of gravity G vertically
inder O. For if its C.G. were at G_1, the weight would
iave a moment round O, causing the body to turn.

Fig. 29
Susper
Body.

EXPERIMENT.—Hang up a sheet of cardboard by a string at one point O and draw the vertical. Repeat for another point of support. The intersection of the two verticals is the C.G.

Similarly when a body rests on a plane, horizontal or inclined, there are two forces acting on it, its weight at the C.G. and the resultant reaction of the plane, and equilibrium these must act in the same straight line. The vert through the C.G. must therefore pass through the area of con of the base of the body with the plane. If the of the body shown in Fig. 30* were at G_1, the b would topple over, but there is equilibrium whe is at G_2. When a body returns to its rest posi after a slight displacement, it is said to be in st equilibrium; if it moves still further away a the displacement its equilibrium is called unste

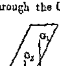


Fig. 30*.

When, like a sphere on a table, it rests indifferently in any posi it is said to be in neutral equilibrium.

EXAMPLES ON CHAPTER III*

1. A particle slides down a smooth inclined plane inclined to the horiz at an angle θ. Show that its acceleration is $g \cdot \sin \theta$.

2. Find graphically and by calculation the resultant of forces of 10 a

CHAPTER IV*

HYDROSTATICS

Fluids.—Solids are substances which can oppose a big resistance to any change in their size or shape. Fluids, on the other hand, are substances, like water and air, which can offer no resistance to shape changes when they take place slowly. For rapid changes internal friction—called also viscosity—comes into play. This accounts for the large horse power required in fast steamers and in aeroplanes. For many purposes, as in the following pages, these frictional effects can be neglected. The term fluids includes both liquids and gases. Liquids differ from gases in being very difficult to compress. A gas will expand until it fills the containing vessel, however large it may be. On account of the absence of friction it is clear, (1) that the surface of a liquid at rest must be horizontal: if it were inclined, the particles would slide down the inclined plane. (2) The pressure on any solid surface in contact with a fluid at rest is normal to that surface. For if it were not so, it could be resolved into components perpendicular and parallel to the surface, and the latter component would cause motion to take place.

Pressure in a Fluid.—If a fluid exerts the same pressure on every cm.² of a surface its pressure is said to be uniform, and is expressed in dynes or gms. per cm.² Strictly this should be called the intensity of the pressure. When it is not uniform, a point is taken and a plane area S is described around it so small in extent that the pressure on it can be regarded as uniform. If p is the total pressure on this area, then p/S is called the pressure at the point. It is evidently the pressure that would be exerted on unit area if it were everywhere of the same intensity as it is over S. The pressures at two points in the same horizontal plane and connected by fluid at rest are equal. For imagine the points A and B (Fig. 31*) to be connected by a smooth tube; if the pressures on the ends were different the fluid would flow to the left or right. Also the pressure at any point is the

is is the same at all points in a vessel. When a U-tube contains quid (Fig. 32* (a)) the pressure at B = pressure at C, and therefore ie height AB = height CD, although the ertical limbs may have different sectional reas. If the U-tube contains two liquids which do not mix, as in Fig 32* (b), let B e the interface where the liquids meet, d_1 nd d_2 the densities of the liquids on the eft and right respectively, and h_1 and h_2 he lengths of the columns AB and CD. Then the pressure at B = the pressure at C (in the same horizontal plane).

$$\therefore\ h_1 d_1 = h_2 d_2$$

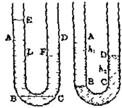


FIG. 32*—Pressure in a U Tube

This result can be used to compare the densities. A U-tube containing liquid can also be used as a pressure gauge For example, suppose the gas mains are connected to the right limb of the U-tube shown in Fig. 32* (a): the liquid is forced downwards in this limb, and upwards in the other. Let it come to rest at E and F, then the gas just balances the column of liquid of height EL, and this accordingly measures its pressure.

Principle of Archimedes.—When a body is partly or wholly immersed in a fluid the pressure on its surface gives rise to an upward thrust and the body apparently loses weight, as, *e g*, in swimming. This apparent loss can easily be found both theoretically and experimentally. Let A (Fig. 33*) represent the body. Imagine it to be removed and the space it occupies filled with the fluid The forces arising from the surrounding fluid are unaltered, but now it is seen that the resultant upward thrust just supports the weight of the volume A. Hence the loss of weight of the original body is equal to the weight of the fluid that would fill the space A. This is known as the principle of Archimedes, after its discoverer. **When a body is immersed in a fluid there is an upward thrust on it of an amount equal to the weight of fluid it displaces, and the body apparently loses weight by this amount.**

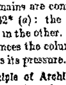


FIG 33*—Principle of Archimedes.

EXPERIMENT—In Fig 34* B is a solid metal cylinder which just fits in the hollow cylinder A. Weigh the two together and then let B be totally immersed in water, as in the fig Balance is destroyed, but it may be restored by filling

A with water, showing that the loss of weight of B is equal to the weight of water it displaces. Corresponding to the upward thrust there is a reaction, and the weight of the water in the beaker is apparently increased. The following experiment shows how much this increase is.

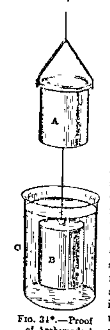


FIG. 31*.—Proof of Archimedes' Principle.

EXPERIMENT.—Find B's loss of weight in gms.; remove it and place the beaker C on the balance; suspend B from a stand with thread so that it is immersed. The beaker gains in weight and its gain will be found equal to B's apparent loss.

In accurate work the weight of a body and the balance weights must both be corrected for the air they displace.

Determination of Specific Gravity.—The strengths of solutions and the purity of substances are frequently tested by finding their specific gravities. If the specific gravity of a ring, said to be gold, is determined and is found to be less than that of pure gold, then it is certain that either the ring is not solid throughout or it is made of some lighter alloy. The specific gravity of a solid is found from Archimedes' principle by weighing the body first in air and then when hanging completely immersed in water. Let these weights be W and W_1 respectively. Then

$$\text{loss of wt. in water} = \text{wt. of water displaced} = W - W_1 \text{ gms.}$$

and
$$\text{S.G.} = \frac{\text{wt. of body}}{\text{wt. of an equal vol. of water}} = \frac{W}{W - W_1}$$

As 1 gm. of water occupies 1 c.cm., it is evident that the volume of the body is $(W - W_1)$ c.cms. This is the most accurate method of finding volumes.

Suppose the same body is now weighed in another liquid, and that its apparent weight is W_2. Then the weight of liquid it displaces is $(W - W_2)$ and, as the weight of the same volume of water is $(W - W_1)$ gms.,

$$\therefore \text{ the S.G. of the liquid} = \frac{W - W_2}{W - W_1}.$$

In case the solid is soluble in water, it must be weighed in some other liquid, of known specific gravity s, in which

it is insoluble. If the weights are W and W_1, as before, the weight of liquid displaced is $(W - W_1)$ gms ; and, as 1 c.cm. of the liquid weighs s gms., the volume of liquid displaced is $(W - W_1)/s$ c cms. But the weight of this volume of water is $(W - W_1)/s$ gms.,

$$\therefore \text{S G. of the solid} = \frac{W}{(W - W_1)/s} = \frac{W}{W - W_1} \cdot s.$$

That is, to find the specific gravity of a body soluble in water we first find the specific gravity referred to some other liquid, then multiply this by the specific gravity of the liquid.

The best method of finding the specific gravity of a liquid is to find in succession the weights of liquid and of water required to fill a small narrow-necked bottle; the S G of the liquid is the ratio of these weights. The specific gravity bottle method is also used when the body is in the form of a powder. W gms of the powder are weighed in the bottle, which is then filled up with water. Let the contents—powder and water—weigh W_1 gms Next the weight of the water alone required to fill the bottle is found; let this be W_2. If it were possible to add the powder to the bottleful of water without any liquid escaping, the combined weight of the contents would be $W + W_2$; actually water escapes and the weight is less than this, W_1 Hence water displaced by powder $= W + W_2 - W_1$,

$$\text{S.G. of powder} = \frac{W}{W + W_2 - W_1}.$$

The methods described above are all used, with additional precautions, in accurate work. The following are much less accurate and, with the exception of the common hydrometer, are used only to illustrate and verify the principles already developed in earlier pages. On account of the accuracy with which weighings can be effected balance methods are usually to be preferred to all others

Nicholson's Hydrometer consists of a hollow cylinder A (Fig. 35* (a)), carrying upper and lower pans C and B. It is suitably weighted to float upright in a liquid. To find the specific gravity of a solid, the hydrometer is floated in water and weights are added to C until a mark P is just in the surface. The solid is then placed in the upper pan and weights W_1 are removed to bring P again in the surface; the weight of the solid is W_1. The body is then removed to the lower pan when, on account of the up-thrust, further weights W_2 must be removed from C to keep the mark in the surface. The weight of water displaced by the solid is W_2, and the specific gravity

required is W_1/W_2. To find the specific gravity of a liquid, the hydrometer is floated first in the liquid and secondly in water, weights being added to C in each case to bring P in the surface. Let W_1 and W_2 be the weights required, and W the weight of the hydrometer alone. From Archimedes' principle the weights of liquid and water displaced are $(W + W_1)$ and $(W + W_2)$, and the volume is the same in each,

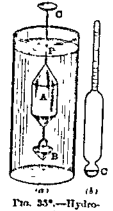


FIG. 35*.—Hydrometers

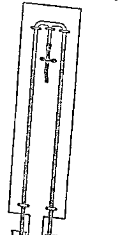
hence the S.G. of the liquid $= \dfrac{W + W_1}{W + W_2}$

The common hydrometer shown in Fig. 35* (b) is frequently used in works' practice. It consists of a hollow glass bulb surmounted by a slender cylindrical stem, on which a scale is engraved; a lower bulb C is weighted so as to make the instrument float in an upright position when placed in a suitable liquid. When placed in a liquid to be tested, the hydrometer sinks until it displaces its own weight of the liquid; the specific gravity of the liquid is then given directly by the scale reading at the surface.

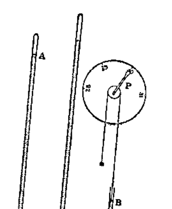
The U-tube method (Fig. 32* (b)) can be used to find the specific gravity of a liquid which does not mix with water. When the liquids are miscible the U-tube is inverted and the liquids are drawn up the limbs (Fig. 36*). The pressure on the liquid in the limbs is the same in each case, viz atmospheric pressure, and this balances the weights of the liquid columns. Hence, as before, $h_1d_1 = h_2d_2$, whence, if d_1 refers to water, d_2 can be found. In this manner the U-tube is called Hare's apparatus.

Pressure of the Atmosphere.—Gases differ from liquids in being less dense, more compressible, and less viscous. If a bulb closed by a stopcock be exhausted of air and weighed, it can be shown that the readmission of the gas increases its weight. [illegible] of dry air at a temperature of 0° Centigrade, and a pressure of [illegible] of mercury (see below), weighs 1·293 gms. Any one [illegible] knows that air is compressible, and that [illegible] fall to earth is a result of the viscosity of [illegible] with a relatively large surface [illegible] larger surface than if they are all [illegible]

o one liquid drop.) As air has weight the atmosphere should oduce a pressure, and this, in fact, it can be readily shown to do. a glass tube 5 mms. in diameter and 80 cms or more in length closed at one end, filled with mercury, and then inverted with its n end under mercury, it is found that a column of the liquid about :ms. long remains in the tube (Fig. 37* (a)) As the pressure at level of B must be the same inside and outside the tube, the sure of the atmosphere just balances that of a column of mercury



'.—Hare's Apparatus.



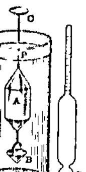
(a) (b)

FIG. 37*.—Barometers

al height AB. If the tube be inclined the mercury runs up ts vertical height is the same as before, since the pressure ends only on the *vertical* depth of this point below A (p. 38*). ght AB is called the height of the barometer. The space ... A is called the Torricellian vacuum; actually it is filled with rcury vapour, but this exerts a very small pressure. The normal ometric height is 76 cms., so that the pressure of the atmosphere sq cm. is that due to 76 c.cms. of mercury. Taking the density mercury as 13 6 gms./cm.^{3}, this is $(76 \times 13\ 6)$ gms. weight =

required is W_1/W_2. To find the specific gravity of a liquid, hydrometer is floated first in the liquid and secondly in water, weig being added to C in each case to bring P to surface. Let W_1 and W_2 be the weights quired, and W the weight of the hydrom alone. From Archimedes' principle the wei of liquid and water displaced are $(W + W$ and $(W + W_2)$, and the volume is the sa each,

hence the S.G. of the liquid $= \frac{W +}{W +}$



(a) (b)
FIG. 35*.—Hydrometers

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cms. long remains in the tube (Fig. 37* (*a*)). As the pressure at
level of B must be the same inside and outside the tube, the
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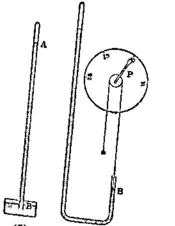
36*.—Hare's Apparatus.



(*a*) (*b*)

Fig. 37*.—Barometers

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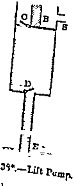
[illegible] the surface of the mercury at D and as the [illegible] or falls, [illegible] means are communicated to the pointer. An aneroid [illegible] barometer is described on p. 3. If water were used instead of mercury in a barometer its height would have to be 13.6 times as large to produce the same pressure. This gives the height of a water barometer as approximately 34 ft. Just as the pressure of a liquid varies with the depth, so does the pressure of the atmosphere decrease as we ascend (see p. 21). The height of a balloon, etc., is calculated from the decreased pressure as measured by a pressure barometer. For this and similar purposes an aneroid barometer is used. This consists of an airtight metal box closed on one side by a thin metal sheet. Variations in pressure produce movements of the sheet which, by suitable gearing, are magnified and communicated to a pointer moving over a dial on which the values of pressures or heights are marked.

Boyle's Law.—The law governing the compressibility of gases is called Boyle's law, after its discoverer Robert Boyle. In the form of an equation it states that $p_1v_1 = p_2v_2$, where p_1 and p_2 are the pressures of a given mass of gas and v_1 and v_2 are the corresponding volumes. For the method of proving the law see p. [illegible]

Lift and Force Pumps and Brahmah Press.—The common pump depends for its action on atmospheric pressure. The piston B (Fig. 38*) carries a valve C which opens upwards; lower down the barrel is another valve D opening in the same direction, and from this a narrower pipe goes to the water reservoir. Normally C and D are closed by their own weight. Let us consider the action from the start:—When B is raised the pressure of the air between C and D is reduced, D opens on account of the air pressure below, and air from the pipe passes into the space CD. The piston is then lowered, the increased pressure closes D and opens C, thus allowing air to escape. These operations are repeated until all

ir is removed from the pipe, and the atmospheric pressure on the eservoir forces water up above D to take its place. At the succeeding lownward stroke this water closes D and escapes above C. On iow raising the piston the water closes C and is carried upwards to scape at S. Evidently DE must be less than the height of the vater barometer—about 34 ft.—otherwise the atmospheric pressure ould not force the water above D. If it be required to raise water a greater height, as with a fire engine, a force pump must be used ne form of this is shown in Fig 39*, where it forms part of a Brahmah ress. The only additional part is the valve B opening upwards. ippose the cylinder L to be full of water When the solid plunger is raised, atmospheric pressure forces water from the reservoir,



. 39*.—Lift Pump.

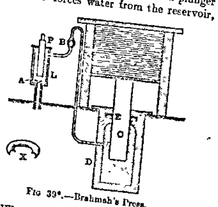


Fig 39*.—Brahmah's Press.

— the valve A, into L. When P descends A is closed; at the me time B is opened by the increased pressure and water is forced wards, whence it may be led away in any direction. In the fig. s shown as being led into a second strong iron cylinder D, forming t of the Brahmah press. A large cylinder C, called the ram, ves up and down in this chamber, suitable packing at E preventing kage. The packing takes the form of a well-oiled leather collar, ed in the groove near E, and of the shape shown at X; the sure of the water on the concave side then presses it strongly nst the metal on either side. When valve B is open the pressure he plunger P is transmitted to the ram, and as the sectional area is is n times that of the plunger, where n is a large number, the upward pressure on the ram is n times that on P. By this s very large pressures can be produced.

Air Pumps.—The simplest form of air pump is the water [illegible] filter pump shown in Fig. 40*. The side tube C leads to the [illegible] to be exhausted, while A is connected with the water main. [illegible] a strong jet of water is forced down A the air surrounding the [illegible] nozzle becomes entangled and is carried away down B, thus [illegible] vacuum can be produced in a vessel joined to C. Even when a [illegible] of air is sent down A a suction effect is produced, as can read[illegible] shown by blowing down it while a tube from C leads into a [illegible] of water. The liquid immediately rises and passes down B. If [illegible]

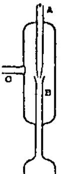


FIG. 40*.—Filter Pump.

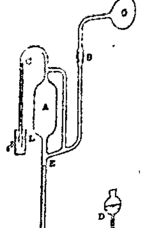


FIG. 41*.—Töpler Air Pump.

be used instead of air we have an illustration of the action of [illegible] steam injector used to force water into boilers. There is also a pi[illegible] and valve air pump, but as this operates in a similar manner to [illegible] lift pump of Fig. 38*, no further description is necessary, except [illegible] say that the narrow tube below D goes to the vessel to be exhaus[illegible] and it is the expansive force of the gas which opens this valve at [illegible] stroke of the piston. If each valve in Fig. 38* opened downwa[illegible] air could be compressed in any vessel fixed on to the end of [illegible] tube DE. This is the action of a bicycle pump, except

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The Siphon.—The siphon (Fig 42*) is a convenient device for mptying a vessel of liquid when taps are not in use. It consists a bent tube with a long and a ort limb. Suppose the tube is led with some of the liquid in the ssel; then the pressure at B, being ual to that at A in the same hori-ntal plane, is that due to the mosphere, plus the weight of a umn of liquid of height DA, and downward pressure at C is greater n the atmospheric pressure owing he liquid column BC. Hence liquid pes and atmospheric pressure forces more up the short limb ake its place. It is clear that the siphon will not work if the ical distance between D and E is greater than the height of a meter made of the liquid in question.

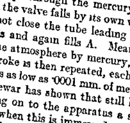

Fig. 42*.—The Siphon.

FIG. 40*.—Filter Pump.

FIG. 41*.—Töpler Air Pump.

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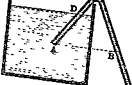
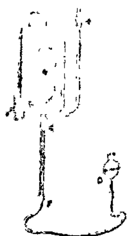


FIG 42*.—The Siphon.

FIG. 40*.—Filter Pump.



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one such, called a Töpler pump, is illustrated in Fig. 41*. The
apparatus is made of glass, except for the rubber connection tube F.
The reservoir D and the vessel L contain mercury, and the length
of each of the tubes FE and LC is slightly greater than the baro-
metric height. The lower end of C is under mercury. The apparatus
to be exhausted is sealed on just beyond the valve B, whose purpose
is to prevent the passage of mercury upwards. When the reservoir
is raised mercury passes up the side tubes at E and closes the
valve, it also flows into A and down the narrow tube C, sweeping
air along and causing it to escape through the mercury in L.
The reservoir is then lowered, when the valve falls by its own weight,
on account of its shape does not close the tube leading down-
d, so that the air in G expands and again fills A. Meanwhile
the lower end of C is sealed from the atmosphere by mercury, some
which rises up the tube. The stroke is then repeated, each time
removing air of volume A. Pressures as low as ·0001 mm. of mercury
can be produced by this pump. Dewar has shown that still lower
pressures can be produced by sealing on to the apparatus a small
vessel containing coco-nut charcoal; when this is immersed in liquid
air it is found that the charcoal rapidly absorbs most gases.

The Siphon.—The siphon (Fig. 42*) is a convenient device for
emptying a vessel of liquid when taps are not in use. It consists
of a bent tube with a long and a short limb. Suppose the tube is
filled with some of the liquid in the vessel, then the pressure at B, being
equal to that at A in the same horizontal plane, is that due to the
atmosphere, plus the weight of a column of liquid of height DA, and
the downward pressure at C is greater than the atmospheric pressure owing
to the liquid column BC. Hence liquid flows and atmospheric pressure forces more up the short limb
to make it flow. It is clear that the siphon will not work if the
vertical distance between D and E is greater than the height of a
barometer made of the liquid in question.

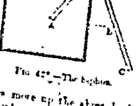
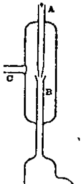
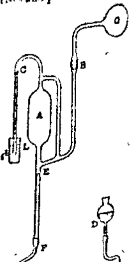


Fig. 42*.—The Siphon.

Air Pumps.– The simplest form of air pump is the ... filter pump shown in Fig. 40*. The side tube C leads to the ... to be exhausted, while A is connected with the water main. ... a strong jet of water is forced down A the air surrounding the ... nozzle becomes entangled and is carried away down B; thus a ... vacuum can be produced in a vessel joined to C. Even when ... of air is sent down A a suction effect is produced, as can ... shown by blowing down it while a tube from C leads into ... of water. The liquid immediately rises and passes down B. ...





he valve corresponding to D is in the tyre. It is evident that with piston and valve air pump the exhaustion cannot be pushed beyond e stage where the pressure of the gas becomes too feeble to raise e valve D. For further exhaustion another type must be used; e such, called a Töpler pump, is illustrated in Fig. 41*. The paratus is made of glass, except for the rubber connection tube F. e reservoir D and the vessel L contain mercury, and the length each of the tubes FE and LC is slightly greater than the barotric height. The lower end of C is under mercury. The apparatus be exhausted is sealed on just beyond the valve B, whose purpose o prevent the passage of mercury upwards. When the reservoir s raised mercury passes up the side tubes at E and closes the e, it also flows into A and down the narrow tube C, sweeping air along and causing it to escape through the mercury in L. reservoir is then lowered, when the valve falls by its own weight, on account of its shape does not close the tube leading down-, so that the air in G expands and again fills A. Meanwhile ower end of C is sealed from the atmosphere by mercury, some uch rises up the tube. The stroke is then repeated, each time ving air of volume A. Pressures as low as ·0001 mm. of mercury e produced by this pump. Dewar has shown that still lower ires can be produced by sealing on to the apparatus a small ontaining coco-nut charcoal; when this is immersed in liquid is found that the charcoal rapidly absorbs most gases.

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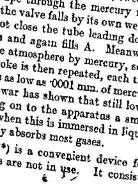

FIG. 42*.—The Siphon.

EXAMPLES ON CHAPTER II*

1 The specific gravity of gold is 19·3, that of silver is 10·5 What i[s the] composition of an alloy of gold and silver whose specific gravity is 17·5? (L.

2 A cu ft of water weighs 1000 oz A man weighing 150 lbs float[s with] 4 cu in of his body above the surface. What is his volume? (L. '91)

3 A piece of lead weighing 17 gms and a piece of sulphur have [the same] apparent weights when suspended from the arms of a balance and immer[sed in] water When the water is replaced by alcohol of density ·9, 1·4 gms are added to the pan from which the lead is suspended to restore equilibrium. (L. '9 ...) [De]termine the weight of the sulphur, the density of lead being 11·333.

4 A bottle whose volume is 500 c cs. is sunk mouth downwards belo[w the] surface of a pond. How far must it be sunk for 100 c.cs. of water to run i[nto the] bottle? The height of the barometer at the surface is 76 cms., and the sp[ecific] gravity of mercury is 13·6 (L. '93.)

5. A flask, which when filled with water weighs altogether 410 gms., [has] 60 gms of a solid introduced, and being then filled up with water weigh[s ...] gms. Find the s g of the solid and the volume of a kgm. of it. (L. '90)

6. Some air is in the space above the mercury in a barometer. The mercury stands at 29 in., the space above the mercury is 4 in. long. The [tube] is then pushed down into the cistern so that the space above the mercury [is] 2 in. long, and now the mercury stands at 28 in. At what height would [it stand] in a perfect barometer? (L. '94)

7. Show that if a piston is moved along a cylinder against a constant pre[ssure] the work done in a stroke is equal to the product of the pressure into the vo[lume] swept out by the piston. (L. '97.)

8. A closed cylindrical vessel 3 ft. in diameter and 1 ft. high is conn[ected] with a vertical tube of 1 sq in. and 10 ft. high from the top of the vessel. [Cal]culate (a) the weight of water that will fill the vessel and tube; (b) the [force] tending to burst off the bottom of the cylinder; and (c) the pressure on [the] bottom. (1 cu. ft. of water weighs 62·5 lbs.) (L. '98.)

9. A man 1·7 metre high changes from the vertical to the horizontal pos[ition]. If the density of the blood be 1·03, calculate the change in blood pressure [in his] head, assuming that it stays constant in his feet. (L. 1900.)

PHYSICS

CHAPTER I

GENERAL PROPERTIES OF MATTER

HE study of Mechanics has shown that a frequent result of the tion of force on matter is the generation of energy, either kinetic or otential. It is found, however, that energy may manifest itself in ther and more complicated forms; although these would probably e reducible to the simpler forms if our knowledge were more omplete. The study of these various forms of energy, their modes f propagation from place to place, and the conversion of one form nto another is the province of Physics. Properly to understand hese, it is found necessary to investigate the properties of the mallest particles of which all matter is built up; consequently the tructure of matter is one of the main problems of Physics. The ypes of energy dealt with in the following pages are those assoiated with Heat, Light, Sound, Magnetism and Electricity; but efore beginning their study it will be convenient to deal with ome of the general properties of matter, which do not fall strictly nder any of the above heads, as these properties are frequently net with in experimental work.

Newton's Law of Gravitation.—Matter may be defined as that which occupies space. This definition does not make any hypothesis as to the structure of matter; in fact, this question is one of the main problems of Physics. In addition to occupying space, all matter possesses mass and is subject to the law of gravitation. This law discovered by Sir Isaac Newton, states that every particle of matter attracts every other particle with a force which is proportional to the product of the masses and inversely as the square of the distance between them. Thus if m and m' are the masses and R the distance between them, the force of attraction

$$F \propto \frac{mm'}{} \quad \text{or} \quad F = k\frac{mm'}{}$$

tered, and each attracts and is attracted by the sun and astronomical bodies. As illustrations of laboratory experiments that have been made to detect and measure the attraction between bodies, the two following may be quoted. Prof. K. Menzel weighed a body on the top of a high building, then brought it to the balance by a long vertical wire, which passed through rooms below, and weighed it again. In the second position it is nearer the earth and should be more strongly attracted, its weight should be greater; this was found to be the case. Secondly, Prof. Poynting attached equal masses to the arms of a balance and found that one was attracted downwards when a large block of lead was placed immediately beneath it. From experiments of this type the value of the constant k in the above equation can be found, for all the quantities except k can be measured. Once this value is known the mass of the earth and of the planets can be calculated.

Elasticity.—When force is applied to a body it may move it as a whole or it may merely alter the relative positions of the particles composing it. In the latter case the size or shape of the body is changed and it is said to be strained. If it tends to recover its original size or shape after the forces are removed the body is said to be elastic. Consider any small plane area in a strained elastic body: there will be attractive or repulsive forces between the particles on opposite sides of the plane tending to move them back to their original positions; the magnitude of this force per unit area is called the stress. Bodies which tend to recover their original volume after a deformation are said to possess **volume elasticity**; those which tend to recover their shape, *e.g.* after a twist, are said to possess **simple rigidity**. So long as their volume is unaltered liquids and gases do not permanently resist change of shape; they have only volume elasticity. Solids, on the other hand, have elasticity of both types. If we consider only alterations of length, the strain is measured by the change in length per unit length; thus if a wire of length L cms. is strained until its length is $(L \pm l)$ cms., the strain is l/L. When the volume varies the strain is measured by the change in volume per unit volume, *i.e.* if the volume V is altered by forces to $(V \pm v)$, the strain is v/V. If the strain exceeds a certain ... which varies with the material, the body is permanently ...d and is incapable of recovering its original configuration ...d in such cases that the elastic limits have been exceeded.

...... strains, within the elastic limits, experiment shows that
he strain is proportional to the stress; hence

$$\text{stress} \propto \text{strain}$$

$$\frac{\text{stress}}{\text{strain}} = E$$

ere E is a constant. This ratio is called the **modulus of elasticity**.
r volume changes E is called the bulk modulus of elasticity; if
are concerned only with variations of length, E is called Young's
dulus. Let a wire of length L and radius R cms. be stretched by
orce of F dynes, and suppose the increase of length is l cms. Across
h section of the wire there acts a force F, hence the stress $= \frac{F}{\pi R^2}$
the strain is l/L, and Young's modulus, which we shall denote
Y, is

$$Y = \frac{F/\pi R^2}{l/L}$$

The ratio l/L is a mere number independent of the units of lengtl
e Y is given in dynes/cm.² If the force is applied by hanging
ht of M gms. to the lower end of a vertically suspended wire

$$F = Mg$$

$$Y = \frac{Mg \cdot L}{\pi R^2 \cdot l}$$

ppose that a body of volume V cms.³ is subjected at every
of its surface to an increase of pressure P dynes/cm.² at right
to the surface, and let v be the volume change produced.
the stress is P, the strain is v/V, and the bulk modulus

$$= \frac{P}{v/V} = \frac{PV}{v}$$

the previous case v/V is a number and the bulk modulus
essed in dynes/cm.²

oke's Law.—Young's Modulus for a Wire.—It has already been
that within certain limits stress and strain are proportional;
erations in length this is known as Hooke's law. This law
hat the linear extension is proportional to the stretching force.
ing balance is a common application of this rule, the extension

Bulk Modulus of Solids and Liquids.—The
easurement of this quantity is difficult and will
ot be dealt with, but the volume elasticity of gases
n easily be determined. Before giving the neces-
ry experiments it will be convenient if some form
standard barometer is described. It is assumed
at the student already knows the principle of the
rometer.

Fortin's Barometer.—To construct a barometer
lass tube 80 cms or more in length is closed at
end, filled with mercury, and its lower, open, end
laced in a vessel of the same liquid. The height
he mercury surface in the tube over that outside
he height of the barometer, and the pressure the
mn produces per cm ² is the atmospheric pressure
convenient to read the height by a scale fixed
he column, but as the pressure varies, mercury
into or out of the cistern and the scale zero
constantly be shifted to coincide with the
ury surface. This difficulty is overcome as
rs in the Fortin barometer. The bottom of
istern is made of wash-leather which rests on
road end of a vertical screw S, while imme-
y above the mercury there is a small ivory
r P (Fig. 2) whose tip coincides with the zero
e scale. When a reading is to be taken the
is turned until the mercury just touches the
n of the pointer, the scale zero is then on the
surface. The length of the column is read by
ier V which is moved over the graduated
y a rack and pinion. Even when the pressure
tant the height varies with the temperature
unt of the expansion of the liquid; in giving
tric heights it is therefore usual to reduce the
d height to what it would be at 0° Centi-
A thermometer T attached to the column
he necessary temperature; the method of calculating the

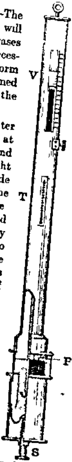

Fig. 2.—Fortin's Barometer.

...ether apparatus, see Barton and Black, "Practical Physics," p. 41.

correction is explained on p. 56. To diminish the effects of sur-
face tension (p. 9), the diameter of the glass tube should be
large, 5–10 mm. at least.

Boyle's Law.—To investigate the elasticity of a gas we have to determine how the volume depends on the pressure. This point may be investigated with the apparatus shown in Fig. 3, due to Dr. Baillie.[1] Two wide glass tubes, A and B, containing mercury are connected by flexible rubber tubing. Each can be moved along vertical iron guides by a string and pulley arrange- ment, and the difference in level of the mercury surfaces is read on a movable scale D. Vessel A is graduated in c cms. from the tap C. This part of the apparatus must first be filled with well-dried air or other gas. With this object some calcium chloride is placed in the funnel above the tap, the latter is opened, and the reservoir B is raised, thus ex- pelling the air from A. The reservoir is nex lowered very slowly and air, dried by the calcium chloride, enters A through the tap. This operation is repeated several times; finally the tap is closed. Suppose mercury stands at the same level in each limb, the pressure of the gas in A is then equal to the pressure at the surface of B, *i.e.* to the atmo spheric pressure, which is measured by the heigh of the barometer. Let this be H cms. at the roo temperature. The volume of the enclosed gas i given by the graduations on A. B is next raise and A lowered until the mercury stands at A′, B The gas pressure P is now $(H + A'B')$ cms. mercury; A′B′ is read on the scale D, and th volume V is found as before. A series of reading is taken in this manner. If B is lower than A, th pressure of the confined gas is (H — difference in level in the two limbs), hence observations can be made with pressures greater and less than that of the atmosphere. Careful experiments of this type have shown that the product (pressure × volume) is constant, provided the temperature does not vary.

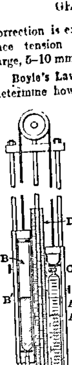

Fig. 3.—Boyle's Law Apparatus.

alled Boyle's law. On the Continent it is known as ...
as the Boyle-Marriotte law. Put in the form of an equation it i

$$PV = \text{constant}$$

if P_1, V_1 represent a second pressure and volume

$$PV = P_1V_1$$

The equation shows that if the pressure on a gas is doubled it
lume is halved. More extended and elaborate experiments hav
own that the law is not strictly true for any gas, but the deviation
e so small in the case of gases like air, hydrogen, oxygen, nitroge
d helium that we shall assume it is obeyed accurately. Oth
ses such as sulphur dioxide, carbon dioxide, and ammonia are mo
mpressible than the law requires; their volume is reduced to le
an half when the pressure is doubled. When we state the volur
f a gas it is clear we must give also the temperature and pressure
which this is measured; the normal temperature and pressu
N.T.P.) are taken as the melting-point of ice, and a pressure
76 cms. of mercury at 0° Centigrade. The above equation may
written in a slightly different form. Let ρ be the density of the
when its pressure is P and volume V. Since density is the mass
unit volume the mass of the gas

$$m = V\rho$$

or

$$V = m/\rho$$

Hence

$$PV = \frac{Pm}{\rho}$$

or

$$\frac{P}{\rho} = \frac{PV}{m} = \text{const.}$$

since m is constant while we deal with the same mass of gas.

Any change which takes place at constant temperature is ca
an isothermal change. Thus Boyle's law gives the isothe
relation between the pressure and volume of a gas.

Isothermal Elasticity of a Gas which obeys Boyle's Law.—I
mass of gas occupy a volume V at a pressure P, and suppose whe
pressure is altered by a small amount to $(P + p)$ that the vo
becomes $(V - v)$, the temperature remaining constant. The inc
in stress is p and the strain it produces is v/V, hence the isothe
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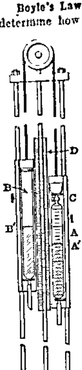

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or

$$V = m/\rho$$

Hence

$$PV = \frac{Pm}{\rho}$$

or

$$\frac{P}{\rho} = \frac{PV}{m} = \text{const.}$$

since m is constant while we deal with the same mass of gas.

Any change which takes place at constant temperature is Isothermal change. Thus Boyle's law gives the isoth lation between the pressure and volume of a gas.

Isothermal Elasticity of a Gas which obeys Boyle's Law.— ass of gas occupy a volume V at a pressure P, and suppose wh ressure is altered by a small amount to $(P + p)$ that the v ecomes $(V - v)$, the temperature remaining constant. The in n stress is p and the strain it produces is v/V, hence the isoth ulk elasticity is

But $\quad (P + p)(V - v) = PV$ from Boyle's law
or $\quad - Pv + pV - pv = 0$

Now p and v can be made as small as we please, hence th product pv can be made so small as to be negligible compared w the other terms of this equation (see p. 40). We may therefi neglect the third term, and

$$pV = Pv$$

or

$$\frac{pV}{v} = P$$

Thus the isothermal elasticity of the gas is equal to its pressu E is usually given in dynes/cm.^{2}, while the pressure P per cm.^{2} measured in cms. of mercury, P must therefore be expressed dynes/cm.^{2} Taking the normal pressure we have to find t weight in dynes of a column of mercury 76 cms. high and 1 c in section. Since the density of mercury is 13 6 and $g = 980$,

$$\begin{aligned} P &= 76 \times 1 \times 13{\cdot}6 \text{ gms./cm}^2 \\ &= 76 \times 13{\cdot}6 \times 980 \text{ dynes/cm.}^2 \\ &= 1{,}010{,}300 \text{ approximately} \end{aligned}$$

This result will be required later.

Kinetic Theory of Matter.—In order to connect and explain t many facts that have been accumulated by experiment, so hypothesis as to the structure of matter is necessary. The kine theory is the one which has proved most fruitful in these respec According to this theory it is supposed that all substances are bu up of very small particles called molecules, just as a handful of sa is composed of fine granules. We may suppose, for simplicity, th the molecules are small spheres; then, even when the spheres are contact, the substance is not continuous, but there are interspa between the molecules which are unoccupied by matter. It further supposed that the molecules are not generally in cont with their neighbours, but that each is moving to and fro in a co tinuous state of agitation, sometimes moving freely, at other times collision with surrounding molecules. In gases the average sepa tion of the molecules is large compared with the dimensions o single molecule, so that considerable freedom of motion is possib In liquids the molecules are supposed to be closer together; th ough a molecule may thread its way through the mass lik ... frequent than in gases. Wit

solids the motion is still more restricted; a molecule now oscillates to and fro round a mean position and is never far removed from it. If the molecules in a solid possessed great freedom of movement the shape of solid bodies would constantly be changing; gases, as we know, occupy the whole volume of the containing vessel, no matter how this is varied. It is easy to see that the molecular separation is greater in gases than in liquids, for when a gas is converted to liquid there is usually a large decrease in volume. Thus 1700 c.cms. of steam, which is water in gaseous form, condense to form 1 c.cm. of water. The theory supposes also that the hotter a body is, the more violent does the molecular agitation become; in fact, in the simplest case of gases, it is assumed that the temperature is proportional to the average kinetic energy of the molecules. An increase of temperature, which means an increase of heat within the substance, thus corresponds to an increase in the kinetic energy of the molecules, and we are lead to make a further supposition, viz. that heat is merely energy. This is a view of which the correctness will be established in later chapters. The kinetic theory gives a qualitative, and in some cases a quantitative, explanation of a number of properties of matter. Thus the pressure of a gas is due to the bombardment of the walls of the containing vessel by the rapidly moving molecules. When a body is compressed the molecules are moved closer together. Again, in solution it is supposed that molecules of the solid become detached and wander away through the molecules of the liquid, so that if we could examine a minute quantity of a solution we should find it far from homogeneous. Similarly porous bodies are those in which the particles are so far apart that the molecules of other substances can find their way into the interstices. Or take diffusion: if a few c.cms. of copper sulphate solution are placed in the bottom of an upright tube and the remainder is filled with water, it is found after some days that the salt molecules have gradually wandered throughout the whole mass of liquid, in spite of the fact that copper sulphate is heavier than water. This process is called diffusion. Gases diffuse more rapidly than liquids on account of the greater freedom of the molecules. Other applications will appear later.

Properties of Liquids.

Surface Tension.—It has been stated that liquids do not permanently resist change of shape when their volume is unaltered; this

only approximately correct and it ceases to be true when most o liquid is in or near the surface layer, as in the case of a thin This air has to be forced into a soap-bubble to make it expand, if the mouth be removed from the pipe stem, the bubble contr and forces the gas out again. In this case the volume actu occupied by the liquid is constant, but the extent of its surfac changed. The liquid film behaves, in fact, like a football bladde it resists an increase in its area and decreases in size directly external force is removed. Numerous experiments can be gi to show that the surface of a liquid acts as if it were a stretch membrane.

EXPERIMENT.—Dip a camel hair brush in a beaker of water; the single ha project in all directions. Remove it from the liquid and the hairs are all dra together as if connected by a stretched membrane. Th experiment and many others are given by Prof. Boys i his book on "Soap-Bubbles."

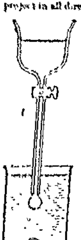


FIG. 4.—Darling's Experiment on the Formation of Drops.

EXPERIMENT.—Place a needle carefully on a wat surface; it rests in a small depression just as a heavi body would do if placed on a sheet of stretched rubber The ability of certain insects to walk on water depend on the same property.

EXPERIMENT.—Make a shallow dish about 2 inches square from fine copper gauze and cover its bottom with a loose piece of paper. Pour water in and then remove the paper; the liquid does not flow out because it must increase its surface before it can escape through the fine holes.

EXPERIMENT.—The formation of a water drop at the end of a vertical tube can be imitated exactly by fastening to a circle of wire a sheet of thin rubber, such as part of a toy balloon. When water is gradually poured on to the rubber it forms a pendant drop very similar to the water drop, and finally contracts like the liquid into a narrow neck before it breaks. When the liquid drops at the end of the tube are large, and if they are caused to form slowly, the similarity becomes more striking. This is done in the next experiment.

Darling's Experiment.—The formation and rupture of a drop can be more easily observed if, by some means the ... the weight of the drop is diminished. A convenient arrangement fo ... in Fig. 4. At 64° aniline has a density equal to that of temperatures just below this it is slightly the denser of the drop of aniline be formed in water at a temperature near 60° it will be supported by the surrounding liquid. The funnel ... lower end, which should have a ... 60°.

...ing the tap slightly a large drop of aniline may be formed at the ...us finally forms a narrow neck and ruptures. The similarity in the ...r of liquid and rubber surfaces then becomes very striking.

...these experiments show that the surface of a liquid acts like ...ched membrane. Imagine a line 1 cm. long drawn on the ..., the tension tends to pull the liquid apart on opposite sides ...line, and the magnitude of the force per unit length is called ...face tension of the liquid. This force is confined to an ex... thin skin of the liquid; hence in the case of a soap-bubble or ...film there is a fully developed surface tension on *each* side. ...he area of such a film is extended more liquid goes from the ...into the surface, but the tension remains constant until the ...rupture is nearly reached. In this respect a liquid surface ...rom a stretched membrane. The ...will perhaps understand more ...the definition given above from ...owing illustration: Suppose we ...wire rectangle (Fig. 5) with a ...n ABCD stretched across it, and ...the side AB can slide along the ...res as guides. On account of the ...tension the film tends to contract ...the wire towards CD; if T is ...ace tension, the force that must ...ied to hold it in position is

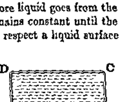


FIG. 5.

T... multiplier 2 comes in since both sides of the ...e to be ta... into account. For water at 0° T is about ...per cm., ... clean mercury it is about 430. The illustration ...n shows a typical effect of surface tension, viz. the tendency ...uid surface has to contract its area unless hindered by other ...Thus raindrops are spherical because that shape has the ...face for the same volume. This circumstance is turned to ...in the manufacture of lead shot. Molten lead is made to ...the top of a tower into water some distance below; during ...it it takes the form of small spheres which rapidly solidify. ...ce tension of mercury is so large that small drops of mercury ...he table are spherical in spite of their weight. The surface ...f oil is less than that of water, hence when an oil drop is ...a beaker of water it is pulled in all directions until it is ...er the entire surface.

[illegible] when [illegible] of the [illegible] as in the case of a thin fil[m] liquid or by an [illegible] the surface layer, [illegible] is stopped, and [illegible] be forced [illegible] the bubble contract[s] [illegible] be removed from the [illegible], the bubble [illegible] the gas [illegible] again. In this case the volume [illegible] occupied by the liquid is constant, but the extent of its surface i[s] altered. The liquid film behaves, in fact, like a football bladder; it [illegible] increases in the area and decreases in size directly th[e] external force is removed. Numerous experiments can be give[n] to show that the surface of a liquid acts as if it were a stretche[d] membrane.

EXPERIMENT.—Dip a camel-hair brush in a beaker of water; the single hai[rs] project in all directions. Remove it from the liquid and the hairs are all draw[n] together as if connected by a stretched membrane. Th[is] experiment and many others are given by Prof. Boys i[n] his book on "Soap Bubbles."

EXPERIMENT.—Place a needle carefully on a wat[er] surface; it rests in a small depression just as a heav[y] body would do if placed on a sheet of stretched rubb[er]. The ability of certain insects to walk on water depen[ds] on the same property.

EXPERIMENT.—Make a shallow dish about 2 inch[es] square from fine copper gauze and cover its bottom wi[th] a loose piece of paper. Pour water in and then remo[ve] the paper; the liquid does not flow out because it mu[st] increase its surface before it can escape through the h[oles].

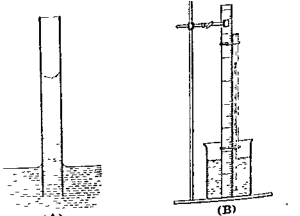
EXPERIMENT.—The formation of a water drop at th[e] end of a vertical tube can be imitated exactly by fasten[ing] to a circle of wire a sheet of thin rubber, such as part [of] a toy balloon. When water is gradually poured on to th[e] rubber it forms a pendant drop very similar to the wat[er] drop, and finally contracts like the liquid into a narro[w] neck before it breaks. When the liquid drops at the e[nd] of the tube are large, and if they are caused to fo[rm] slowly, the similarity becomes more striking. This [is] done in the next experiment.

FIG. 4.—Darling's Experiment on the Formation of Drops.

Darling's Experiment.—The formation and ruptu[re] of a drop can be more easily observed if, by some mea[ns] the effective weight of the drop is diminished. A convenient arrangement f[or] doing this is shown in Fig. 4. At 64° aniline has a density equal to that [of] water, while at temperatures just below this it is slightly the denser of t[he] two; hence if a drop of aniline be formed in water at a temperature near 6[4°] most of its weight will be supported by the surrounding liquid. The fun[nel] shown in Fig. 4 contains aniline, and its lower end, which should have [illegible]

EXPERIMENT.—Place a film of water on a glass plate and let a single d[rop] of alcohol fall on it. The surface tension is diminished and the film is pul[led] away in all directions.

Methods of measuring Surface Tension.—*1st method :* When [a] glass tube of fine bore is held vertically with its lower end in a liq[uid] which wets it, it is found that the liquid rises in the tube to a defin[ite] height which depends on the nature of the liquid and on the intern[al] diameter of the tube. The upper end of the column is hemispheric[al] with its convex face downwards, as in Fig. 6, A. This upper surfa[ce]



(A) (B)

FIG 6.—Rise of Water in a Capillary Tube.

clings to the glass and by means of its surface tension it suppo[rts] the weight of the column below it; this gives a method of find[ing] the surface tension T. Let d be the density of the liquid, h t[he] height which it rises, and R the radius of the tube. Then the surf[ace] tension acts along a length $2\pi R$ and the upward force is $2\pi R \cdot T$ dyn[es]; the weight of liquid it supports, in dynes, is $\pi R^2 hdg$.

hence
$$2\pi RT = \pi R^2 hdg$$
$$T = \frac{Rhdg}{2}$$

To carry out the experiment the glass tube is thoroughly clea[ned] and washed out with the liquid; it is then fixed to a graduated sc[ale]

and placed in a vertical position in the liquid, as shown in Fig. 6, B. The quantity h can thus be found. The diameter of the bore is next measured with a microscope or by other means, and the density of the liquid is found with a specific gravity bottle.

Mercury may be taken as typical of those liquids which do not wet a solid placed in contact with them; these liquids do not rise in capillary tubes like the water in the last experiment.

EXPERIMENT.—Push a capillary tube into mercury; it will be found that the liquid is lower inside the tube than outside, exactly the reverse of what happens with water. It will also be noticed that the surface is convex towards the air.

EXPERIMENT.—Pour mercury into a glass U-tube one limb of which is wide while the other consists of a fine capillary. The liquid surface is lower in the narrow tube. If the bore is very fine a considerable pressure will be required to force the mercury along it. An instance of this is given in the next chapter in connection with the filling of a thermometer.

The rise of a liquid up blotting paper is due to surface tension; the interspaces between the fibres form a large number of capillary tubes through which the liquid ascends. The ascent of a liquid through a lump of sugar is due to a similar cause.

2nd method. In this method the pull due to surface tension is determined directly by means of a balance. A plate of glass, such as a microscope slide, is fixed in a strip of wood and hung with its plane vertical from one arm of a balance over a vessel of water, as in Fig. 7. An equal weight is placed in the other pan. The vessel is raised until the water just touches the glass, when the surface tension pulls down the arm. The height of the liquid and the weights in the other pan are altered until the lower edge of the plate is just in the liquid surface when the beam of the balance is horizontal. Let l be the length and c the thickness of the plate, m the additional weight required to balance the downward pull of the liquid. Taking into account each side of the glass, the surface tension acts on a length $2(l+c)$. Hence the pull is

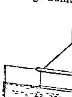


FIG. 7.—Method of finding Surface Tension by Weighing.

$$2(l+c)T$$

whence T can be found.

[illegible]

and

[illegible]

When the plates are used a mercury balance is unnecessary.

Diffusion.—We have already given an example of diffusion and have shown how the process is to be pictured according to the kinetic theory. Let us return to this example (p. 9). Imagine a horizontal plane drawn across the tube just above the copper sulphate some time after the diffusion has begun. As the salt is more concentrated below than above the plane more of its molecules will wander across in an upward than in a downward direction. It can be shown that the excess is proportional to the difference in the concentrations of the salt immediately above and below the plane. The rate at which diffusion proceeds depends on the velocity of the molecules, it accordingly takes place more rapidly as the temperature rises and is quicker for light than for heavy molecules. Owing to their greater freedom of movement gases diffuse much more rapidly than liquids.

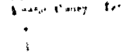


FIG. 8.—Diffusion of Gases.

EXPERIMENT.—The glass tube A (Fig. 8) is closed at its upper end with a plug of plaster of Paris. Fill it with hydrogen and invert it with its open end under the surface of water. Hydrogen is lighter than air and therefore diffuses more rapidly; owing to this the gas escapes through the porous plug more rapidly than air can enter and water rises in the tube. If the tube contains air and is surrounded by an atmosphere of hydrogen the latter gas diffuses inwards and increases the pressure.

The time required for a given volume of a gas to diffuse through [illegible], i.e. the rate of diffusion, varies inversely as the square root of the density of the gas.[1] When a mixture of two gases is passed through a porous tube, such as the stem of a clay pipe, the lighter gas diffuses most rapidly through the walls, hence the mixture

[1] This is Graham's law.

[illegible]

[illegible]

1. [illegible]

2. [illegible]

3. [illegible]

4. [illegible]

5. [illegible]

6. [illegible] (L. '9[illegible])

7. A narrow glass tube is closed at one end and contains air which is shut in from the atmosphere by a long thread of mercury. Show how to find the height of the barometer from observations of the lengths of the air and mercury columns when the tube is (1) horizontal, and (2) vertical.

8. [illegible] in the space above the mercury in a barometer of which the tube is [illegible]. When the mercury stands at [illegible] in. in the tube the space above the mercury is [illegible] in. long. The tube is then pushed down into the [illegible] so that the space above the mercury is only [illegible] in. long, and now the mercury stands at [illegible] in. At what height would it stand in a perfect barometer? [illegible]

9. State the laws of diffusion of gases through a plug of porous material. A mixture of hydrogen and oxygen in equal proportions is contained (1) in a vessel in which there is a porous plug, (2) in a vessel in which there is a [illegible] say, 1 mm. in diameter; the mixture is allowed to escape into a vacuum. [illegible] will the proportion of hydrogen and oxygen be affected, if at all, [illegible] has been going on for a short time? (L. '9[illegible])

[illegible]

[illegible]

[illegible]

An [illegible] used for measuring temperature is [illegible] thermometer. An important part of the study of heat is conc[illegible] with temperature measurement or thermometry. When the tem[illegible] ture of a body changes it is assumed that it has lost or gained [illegible] Thus if a block of hot copper is placed in a beaker of cold wate[illegible] copper becomes cooler and the water hotter; these change[illegible] ascribed to a transfer of heat from the hot to the cold body; generally, as later experiments will show, heat tends to [illegible] from places where the temperature is higher to those where [illegible] lower.

A system of thermometry in order to be scientifically useful [illegible] enable two observers at different places to measure temper[illegible] [illegible] comparable with one another. Thus if obser[illegible] [illegible] substance melts at a temperature of 50°, as record[illegible] [illegible]eter, observer B should know what temperatur[illegible] [illegible] to on his thermometer, although the two instru[illegible]

may not have been compared directly, and may be alt[illegible] different in construction. To construct such a temperature sca[illegible] must be some zero of temperature that can be easily repr[illegible] and also some standard temperature difference in terms o[illegible] any other temperature interval can be expressed. Before [illegible] is this is arranged let us examine first some of the effects p[illegible] a temperature change; we shall then be in position to apply one or more of them to temperature measurement.

General Effects produced by Heat. When [illegible] are heated they generally increase in length, area and volume. The well-known experiment with Gravesande's ring illustrates this. A metal ball is made of such a size that it just passes through a round hole in a sheet of metal when both are at the same temperature. If the ball is heated it can no longer pass through, showing that it has expanded.

Liquids and gases also expand when their temperature is raised, but the observations are here complicated by the expansion of the vessel in which they are contained. This is shown in the following experiment.

Fig. 10 [illegible]

Experiment 32. A small glass flask with [illegible] [illegible]

[illegible]

[illegible] ... A very slight [illegible] ... produce a big expansion, showing that gases are more expansible [illegible] liquids.

When a gas is heated while the volume is kept constant, it [illegible] found that its pressure increases. This can be demonstrated co[illegible]veniently by the apparatus shown [illegible] Fig. 34, p. 65. The bulb A, which contai[ns] air, communicates by rubber tubing w[ith] the mercury reservoir B. When the air [is] heated it tends to expand, but its volu[me] can be kept constant by raising the reserv[oir] until the mercury in the left-hand li[mb] returns to its original position. T[he] mercury in B then stands higher than in [A,] and the difference in levels shows by ho[w] much the pressure of the gas exceeds th[e] atmospheric pressure.

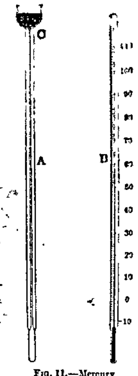


Fig. 11.—Mercury Thermometer.

The electrical properties of bodies a[re] also altered by heat, but the discussion [of] these is deferred for the present.

Any of the above changes produced b[y] heat might be used as a means of measurin[g] temperature; at present we will consid[er] the expansion of a liquid contained i[n] a glass bulb. The liquid most generall[y] chosen is mercury.

Mercury Thermometers.—Our object, i[n] the first place, is to construct an apparatu[s] which will enable us to observe readily th[e] expansion of mercury. The usual form [of] the thermometer is shown in Fig. 11, [B.] It may be constructed as follows: Into [a] carefully cleaned piece of capillary gla[ss] tube is introduced a column of mercur[y] about 2 cms. long, and the length of this [is] carefully measured by means of a micr[o]scope at different parts of the tube. If the bore is unifor[m it] will be constant. Having by this means selecte[d a] piece of tubing, bulbs of convenient size are blow[n as sh]own in Fig. 11, A. The cylindrical bulb at the low[er]

end has the great advantage over a spherical one that it can be readily passed through corks. Dry, clean mercury is placed in the upper reservoir, and the bulb is slightly heated so as to expel some air. It is then allowed to cool; this reduces the pressure inside and mercury runs into the bulb. This procedure is necessary because, owing to the effects of surface tension, the mercury will not flow down the stem except under pressure. By alternate heating and cooling the whole is filled with mercury and this is finally boiled to expel the last traces of air and moisture. It is now heated to a temperature rather above the highest it is to be used to measure, and the glass at C is sealed off in a small blowpipe flame. By this process it is ensured that no air is present above the mercury, so that it can expand freely, and the stem being closed the mercury surface will not become fouled with dirt or moisture. If we now mark the positions of the end of the mercury thread which correspond to two fixed temperatures that can easily be reproduced, and divide the distance between them into a number of equal divisions, the instrument could be used as a means of measuring temperature. The interval between the two fixed temperatures is called the fundamental interval. Let us divide this into 100 equal divisions and number them, starting below, from 0 to 100. If when the thermometer is put into water the mercury stands at the 15th division, it could be said that the temperature is 15 degrees, meaning that the difference between the lower fixed temperature and the temperature of the water is 15/100 of the fundamental interval. In this way, temperatures read by different thermometers would be directly comparable, provided, of course, that the same fixed points were used. We now proceed to show how the fixed points are chosen.

Determination of the Fixed Points.—Thermometers filled as above are usually left for some weeks in order that certain irregularities may disappear which are caused by the heating necessary to blow the bulb. If one of these thermometers is placed in pure melting ice on successive days it is found that the mercury stands at the same point on each occasion. Melting ice therefore provides us with a standard temperature which can readily be reproduced. In a similar manner, if the thermometer is closely surrounded by steam coming from boiling water, it is found that the mercury always stands at another fixed point on the stem, provided that the height of the barometer is the same in each experiment. Accordingly the temperature of melting ice, and of the steam coming from water

Fig. 12.—Apparatus for fixing the Steam Point.

[illegible]

Experiment.—Take two large [illegible] flasks and half fill one with distilled water, the other with a solution of salt in water. Make each liquid boil and place a thermometer in the neck of the first so that the bulb is just above the water. Note the temperature, then transfer the thermometer to a similar position in the other flask; it will be found to show practically the same temperature in each case, and the experiment will be still more satisfactory if the flasks are replaced by the vessel shown in Fig. 12. Now put the thermometer bulb right into the liquids, the temperature is probably higher in each case than it was before, and the temperature of the salt solution may be considerably higher.

For this reason thermometers are immersed in the *steam* of boiling water when the upper fixed point is determined, since the temperature is then independent of any impurities dissolved in the liquid. If a thermometer is placed in ice, and salt is then added, the temperature falls considerably; pure ice must therefore be used for determining the lower fixed point.

A simple apparatus for determining the steam point is shown in Fig. 12.[1] Water is boiled in the copper vessel A, and steam, after c[illegible] as shown by the arrows, escapes by the vent B, which [illegible]le, otherwise the rapid production of steam may crea[illegible]

[1] [illegible] also Barton and Black, "Practical Physics," p. 51.

a pressure in excess of that outside; the water gauge D sho whether this is the case or not. The thermometer is placed in t inner tube C with the exposed end of the mercury thread just abo the cork. As this tube is surrounded by steam the whole arran nent evidently secures that its walls are at the same temperature the steam and the thermometer, and there is no tendency for heat to pass from the bulb to the outside by radiation (p. 116). Let us assume that the barometer stands at 760 mm. After a quarter of an hour the temperature becomes steady and the position of the mercury thread is marked on the stem. The lower fixed point is now determined at once. The thermometer is placed in shavings of melting ice[1] contained in a glass funnel (Fig. 13). The mercury at first falls rapidly, then more slowly, and may finally rise by a small amount. The lowest position it reaches is marked. The interval between the two marks is then divided into 100 equal parts, and the divisions may be extended above and below to enable us to read temperatures above 100° C. or below 0° C. A small bulb is usually made at the top of the stem to diminish the risk f breakage, if, by accident, the apparatus is eated too strongly.

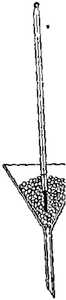


FIG 13 —A tus for find Ice Point.

As mercury freezes at −40° C. a mercury hermometer cannot be used below about −30°; or lower temperatures other liquids, such as alcohol or toluene, must be employed. These ermometers should be compared with some m of standard thermometer. Mercury boils 350°, but before this temperature is reached consid aporation takes place which reduces the amount of liquid em; there is also a tendency for bubbles of mercury vap rm and break the thread. For these reasons mercury th eters cannot be used to measure temperatures accurately bove 230° C. Some thermometers, intended for high temper ontain an inert gas like nitrogen in the upper part of the

[1] A convenient machine for breaking up ice into fine shavings is Messrs. Avery.

This, by its pressure, raises the boiling point (Chap. VIII.) and hinders the breaking of the column by bubbles of vapour. The bulb must be made strong to withstand this pressure.

In the determination of the upper fixed point it was assumed that the water was boiling under a pressure of 760 mm. of mercury; if this is not the case the temperature of the steam will not be 100° C. and a correction must be made. Experiments showing how the boiling point varies with the pressure are described in Chap. VIII. Let us suppose the pressure is 750 mm.; from tables giving the boiling point it is found that under this pressure water boils at 99·63° C. The distance between the marked points, supposing the ice point accurately determined, corresponds therefore to 99·63°. We may calculate by simple proportion where the mark corresponding to 100° should be placed and make it accordingly.

Errors in Mercury Thermometers.—Even when a thermometer has been constructed as described above, certain corrections have to be applied when it is used for accurate work.

Zero Correction.—EXPERIMENT.—Test the zero of a common thermometer that has not been used for some weeks, by means of the apparatus shown in Fig. 13. The mercury will usually stand at a point above the zero of the scale. If it is kept for some hours at a temperature considerably below 0° the size of the bulb will be increased. This is due to a slow contraction in the volume of the bulb, usually called the secular change, which may take years to complete.

EXPERIMENT.—Keep the thermometer at 100° for 30 mins. and again determine the freezing point. The mercury stands lower than it previously did owing to a temporary increase in the volume of the bulb. Experiment shows that the zero determined in the latter case is fairly constant on different days; it is for this reason that the lower fixed point is found immediately after the steam point. These zero changes are due to the fact that, after being heated, glass takes a long time, extending in some cases to years, to regain its initial volume. They are largely reduced by the use of special kinds of glass. It has recently been found that they are entirely absent if the envelope is made of fused silica. Evidently if the zero has risen 0·1° every temperature read on the thermometer will be too high by this amount.

Other errors which have to be considered are due to (1) changes in the size of the degree owing to the distance between the two fixed points becoming slightly different from 100 divs.; (2) Irregularities in the bore of the tube; (3) Changes in the volume of the bulb caused by pressure, either internal or external; (4) The mercury [illegible] at a different temperature from that in the bulb, [illegible] correction.

[illegible] a long and laborious series of [illegible]

too complicated for the present book. To test No. 4 the following experiment may be performed :—

EXPERIMENT.—Place a thermometer in the boiling point apparatus (Fig. 12), leaving the stem from 50° to 100° exposed above the cork ; note the temperature when it becomes steady. Now push the thermometer through the cork until merely the top of the mercury is visible. The reading will be slightly greater because the mercury between the 50° and 100° divisions has become hotter and has expanded.

Let t_1 be the temperature of a bath as read on a thermometer when n divisions of the thread are exposed above the surface ; let t_2 be the mean temperature of the exposed column, and σ the apparent coefficient of expansion of mercury in glass (p. 48). Then the true temperature of the bath $t = t_1 + n\sigma(t_1 - t_2)$. For a proof of this formula see p. 57. This correction is uncertain since t_2 is not known accurately ; it should be made small by immersing the thermometer as far as possible ; σ may be taken to be 0·00015. If $t_2 = 20°$ and $t_1 = 99·4°$ in the above experiment, the true temperature is 100°.

EXPERIMENT.—To test the trustworthiness of the correction immerse the thermometer in the last experiment to different depths, read t_2 by another thermometer placed near the middle point of the exposed stem, and calculate t. Compare the results with the reading obtained when all the stem is immersed.

Nearly all the corrections given above can be found by direct comparison with a standard thermometer. The two instruments are placed in the same bath and their corresponding readings observed at different temperatures. A table is now drawn up showing the amount that must be added to or subtracted from the reading of the incorrect thermometer to make its readings agree with those of the standard. From these a curve can be plotted as in Fig. 14, which enables us to determine the correction for any reading. The reading of the incorrect thermometer is shown on the horizontal line, and the amount to be added or subtracted to get the true temperature is indicated by the vertical distance of the curve from the axis of temperature. Thus if the reading is 80° the curve shows that 0·18° must be added.

Thermometers for Special Purposes.—*Maximum and Minimum Thermometers.* For some purposes thermometers are required which will show the highest or lowest temperature to which they have been subjected during a certain time. ...

a maximum thermometer and may take various forms. In Fig. 15 a is a maximum and b a minimum thermometer.

The bulb of a is filled with mercury, Q is a dumb-bell shaped piece of coloured glass. We have seen, p. 10, that the surface of a liquid offers a resistance to rupture; if the mercury expands it therefore pushes Q along. When cooling takes place the index is left in position and shows the maximum temperature reached. The liquid in b is coloured alcohol, and the glass index P, in a manner similar to Q above, is pulled to the right when the temperature falls. If the temperature rises afterwards P is left in position showing the minimum temperature experienced by the thermometer. The

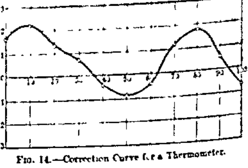


FIG. 14.—Correction Curve for a Thermometer.

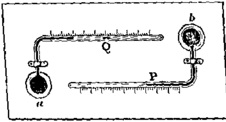
instruments are set ready for use by shaking the index into contact with the surface of the liquid.

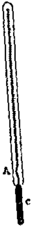
Fig. 15, c, shows a clinical thermometer used by doctors. At A the bore is constricted, the mercury expands past this, but when cooling takes place the liquid column breaks at the constriction and the further end of the thread shows the maximum temperature.

Six's maximum and minimum thermometer, largely used by gardeners, is shown in Fig. 15, d. The bulbs, A and B, containing alcohol freed from air, are separated by a column of mercury E. Two dumb-bell shaped iron indexes, D and C, are pressed lightly against the glass by weak springs. If the temperature rises the liquid in A expands and D is pushed upwards; a fall in temperature similarly causes an upward movement of C. The springs hold each in its extreme position when the mercury retreats. A magnet

e used to bring the indexes into contact with the mercury when the thermometer is set for use.

til the invention of electrical thermometers the differential





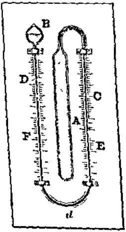


Fig 15.—Maximum and Minimum Thermometers.

nometer (Fig. 16) was very largely used for measuring ure differences, especially in radiation experiments. The bulbs contain air, C is a small index of coloured liquid.

The bulbs are first put in communication with each other by the tap B, which is then closed; if now one bulb be slightly heated the air in it expands and the index moves. Other thermometers will be described later when the changes produced by heat have been further studied.

Other Thermometric Scales.—The division of the fundamental interval into 100 degrees is not the only system used. In the Fahrenheit thermometer the melting point of ice is called 32°, and the boiling point of water 212°, so that the fundamental interval is divided into 180°. This is the thermometer in common use in England for non-scientific purposes; it is also frequently used by engineers and metallurgists.

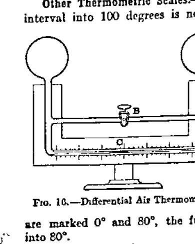


FIG. 10.—Differential Air Thermometer.

On the Réaumur scale, which is in use on the Continent, the fixed points are marked 0° and 80°, the fundamental interval being divided into 80°.

Suppose we require to convert from one scale to another. Let the temperature of the same bath, as read by Centigrade, Fahrenheit and Réaumur thermometers be C, F, and R respectively. The distance of the end of the mercury thread from the lower fixed point, measured in degrees, is C, (F − 32), and R; this distance must evidently be the same fraction of the fundamental interval in each case; hence

$$\frac{C}{100} = \frac{F-32}{180} = \frac{R}{80}$$

or

$$\frac{C}{5} = \frac{F-32}{9} = \frac{R}{4}$$

EXAMPLE.—Convert 80° F. into degrees Cent. In the above equation put F = 80, then

$$\frac{C}{5} = \frac{80-32}{9}$$
$$C = 26{\cdot}6^\circ$$

EXAMPLES ON CHAPTER II

1. What is meant by a scale of temperature, and on what does the definition of any particular scale depend? (L. '07.)

2. Convert the following temperatures from the Centigrade to the Fahrenheit scale: 80°, − 40°, − 273, 1000°. Also find at what temperature the two scales agree.

3. The temperature of a living room is 66° F., that of the blood is 98° F., and the temperature on a hot summer's day is 83° F. Find the corresponding readings on the Centigrade scale.

4. A thermometer which has been tested in the usual way is sunk to its 20° mark in a liquid and reads 90°. The mean temperature of the rest of the stem is 25°. Find the true temperature of the liquid, the coefficient of expansion of mercury in glass being 0·00015. (L. '08.)

CHAPTER III

CALORIMETRY AND SPECIFIC HEAT

Heat as a Quantity.—Up to the present we have considered various effects caused by changes of temperature without inquiring whether it is possible to measure the quantities of heat involved. Let us now consider this point. When 100 gms. of water at a temperature of 40° are mixed with an equal quantity at 20°, the temperature of the mixture is very approximately 30°. The hot water has lost heat and the cold water has gained it. When 300 gms. of hot water whose temperature is 40°, are poured into 100 gms. at a temperature 20°, the resulting temperature is 35°. The cold water has gained more heat than in the first experiment; we are thus led to the idea of different amounts of heat and therefore of heat being a measurable physical quantity. The first point to be settled is what shall be taken as the heat unit. Any physical change that heat produces may be used to define this; it is merely a matter of convenience in measurement that influences our choice. Thus the heat required to melt one gram of ice might be taken as the unit; we should then be justified in assuming that it takes two units to melt two grams and m units to melt m grams. It is, however, found more convenient to define the unit quantity of heat as that required to raise the temperature of 1 gm. of some standard substance, such as water, through 1°. To raise 10 gms. through 1° will then require 10 units, but we are not justified in assuming that 50 units must be supplied to heat 1 gm. through 50°, for the heat necessary to raise the temperature from 10° to 11° might differ from that required to heat the same mass from, say, 40° to 41°. It must therefore be specified at which part of the temperature scale the 1° interval is to be taken. Although there is ... agreement on this point, that most usually chosen is [illegible] C. The unit of heat is then defined as the quantity of heat ... to raise the temperature of 1 gm. of water from 15° to 16° ...

This unit is named the calorie or therm. When a gram of water cools from 16° to 15° it gives out, or loses, one calorie. It has been stated above that the heat lost by 100 gms. of water in cooling from 40° to 30° is just capable of raising the temperature of an equal mass from 20° to 30°. This, if strictly true, would prove that the average quantity of heat required to change the temperature of a gram of water by 1° is the same between 20°–30° as between 30°–40°. More accurate experiments, however, show that this is not quite true, but as the difference is very small, even when other temperatures are taken, it will be assumed in the following pages that the addition of one calorie will change the temperature of one gram of water by 1°, no matter what is its initial temperature, so long as it is between 0° and 100°. To raise the temperature of m gms. through 1° will then require m calories, and to heat the same mass from t_1° to t_2° $m(t_2 - t_1)$ calories must be supplied. This also is the number of units of heat given out by m gms. in cooling through the same temperature range. The measurement of quantities of heat is called calorimetry, and the vessels in which the measurements are carried out are called calorimeters.

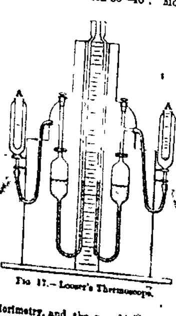


Fig. 17.—Loeser's Thermoscope.

Thermal Capacity and Specific Heat.—The number of calories required to change the temperature of a body by 1° is called its thermal capacity. It varies with the mass of the body and depends also on the nature of the substance. A lecture experiment shows this as we shall find the apparatus to be used very convenient at a later stage a full description of it is given here. Fig. 17 illustrates Loeser's thermoscope; it consists of two separate air thermometers, but into the bulb of each there is fused a graduated test-tube

CHAPTER III

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[illegible] Each bulb [illegible] water [illegible] When a body becomes hot [illegible] substance expands [illegible] the temperature change [illegible] in the [illegible] tube [illegible] (Fig. [illegible]).

EXPERIMENT.—Fill the bulbs [illegible] with cold water. [illegible] pieces of copper and lead in a beaker of boiling water; when they have [illegible] the temperature of the [illegible] transfer them quickly one into each bulb so that they are completely immersed. The hot bodies lose their heat to the water, which in turn heats the [illegible] bulbs. It will be found that the rise in temperature is roughly three times greater in the bulb containing the copper than in the other, showing that when equal masses of these metals are [illegible] through approximately the same range of temperature, copper emits three times as much heat as lead.

EXPERIMENT.—Vary the experiment by putting equal masses of turpentine and water in the two tubes and then drop into them equal masses of copper at about 100°. The bulb containing turpentine rises in temperature about twice as much as the other, showing that this liquid requires less heat to raise its temperature than water does, or its thermal capacity is less, taking equal masses.

The number of calories required to change the temperature of one gram of a substance by 1° is called the specific heat of the substance at the given temperature. It follows from this definition that the specific heat of water is unity at 15°, since one calorie is necessary to change the temperature of 1 gm by 1°, according to what has been said on p. 31, we shall assume that it is unity at all temperatures between 0° and 100°. The first of the two experiments given above shows that the specific heat of copper is greater than that of lead, and the second that turpentine has a less specific heat than water. From the definition, if the specific heat of a substance is s (assumed constant at all temperatures), we have that

To heat 1 gm. of it through 1° requires s cals.
,, m gms. ,, ,, 1° ,, ms cals.
,, m ,, ,, from $t_1°$ to $t_2°$,, $ms(t_2 - t_1)$ cals.

This expression is a fundamental one in calorimetry. When put into words it tells us that :—When the temperature of a body changes the number of calories absorbed or emitted is obtained by multiplying the mass of the body by the specific heat and by the

temperature change. From the definition given above it follows that the thermal capacity of a body is $m \times s \times 1 = ms$.

Measurement of Specific Heat.—The following example will best illustrate how the specific heat of a solid can be found by what is called the method of mixture.

EXAMPLE.—*A block of copper weighing 93·5 gms. was heated in boiling water to 100°. It was then dropped into a calorimeter containing 200 gms. of water at 16·4°. The temperature of the mixture was 20°; find the specific heat, s, of the copper.*

We have to express that all the heat given out by the copper goes into the water in the calorimeter.

The heat absorbed by the water $= 200(20 - 16{\cdot}4) = 720$ cals.

Heat lost by the copper $= 93{\cdot}5(100 - 20)s = 93{\cdot}5 \times 80 \times s$ cals.

and these quantities are equal,

$$\therefore 93{\cdot}5 \times 80 \times s = 720$$

$$s = 0{\cdot}096$$

There are several sources of error in this experiment which must be eliminated in accurate work: (1) The metal cools while it is being transferred from the hot to the cold water; it also carries with it some of the hot liquid so that all the heat given up does not come from the copper; (2) Part of the heat emitted by the copper goes to raise the temperature of the calorimeter itself; (3) Directly the calorimeter and its contents become hotter than surrounding bodies they begin to lose heat by conduction and radiation (p. 115).

To eliminate the first error as far as possible the substance must be heated without coming in contact with the hot liquid, and a more convenient method of transferring it from the heater must be employed. The apparatus described below shows how this is done. The second source of error can be allowed for in the calculation, for if m_2 is the mass of the calorimeter and s_2 the specific heat of its material, the heat absorbed by the calorimeter alone when its temperature is raised from $t_1°$ to $t_2°$ is $m_2 s_2(t_2 - t_1)$ cals. The heat absorbed by the cold liquid is $m_1 s_1(t_2 - t_1)$, if m_1 is its mass and s_1 its specific heat. The heat emitted by the hot body is similarly $Ms(T - t_2)$, if T is its initial temperature and t_2, as before, the temperature of the whole calorimeter after mixture. Equating the heat emitted to the heat absorbed, we have the equation

$$Ms(T - t_2) = m_1 s_1(t_2 - t_1) + m_2 s_2(t_2 - t_1)$$

from which s can be found if s_1 and s_2 are known. The last term is

(A in Fig.). Each bulb communicates through rubber tubing with one of two U-tubes containing coloured water for an index. The tubes are open at their further ends to the external air. By means of taps either bulb may be put directly in communication with the atmosphere when necessary. When a bulb becomes hot the air it contains expands, and the temperature change is proportional to the movement of the corresponding liquid index in the narrow limb of the U-tube, as in the mercury thermometer (Chap. II.).

EXPERIMENT.—Half fill each test-tube with cold water. Suspend equal masses of copper and lead in a beaker of boiling water; when they have taken up the temperature of the bath transfer them quickly one into each test-tube so that they are completely immersed. The hot bodies lose their heat to the water, which in turn heats the thermometer bulbs. It will be found that the rise in temperature is roughly three times greater in the bulb containing the copper than in the other, showing that when equal masses of these metals cool through approximately the same range of temperature, copper emits three times as much heat as lead.

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[illegible] value of [illegible]

[illegible] Hence [illegible] may be taken as [illegible] the mass of water [illegible] which has the same thermal capacity as the calorimeter [illegible] last paragraph [illegible] water equivalent of the calorimeter. [illegible] for example, [illegible] temperature of the calorimeter 1°, the quantity of heat [illegible] the temperature of [illegible] gms. of water by 1°; the [illegible] the temperature of [illegible] gms. When we calculate the heat [illegible] water equivalent is therefore [illegible] gms. [illegible] the water it contains, we [illegible] absorbed by the calorimeter and the water [illegible] and multiply the [illegible] to the weight of the water and [illegible] give the heat [illegible] rise in temperature. The product will give the [illegible] If the whole of the [illegible] not be the calorimeter and its contents. [illegible] water equivalent [illegible] calorimeter [illegible] from t_1' to t_2' the water [illegible] in which it is to be found experimentally under the conditions in which it is assumed that the [illegible] used. The following experiment, in which it is assumed that the [illegible] [illegible] is to be used two-thirds filled, will show the method.

EXPERIMENT.—The weight of the empty calorimeter was 71·9 gms. [illegible] cold water was poured in and a reweighing showed that the amount added [illegible] 81·7 gms. The liquid was well stirred, so as to take up the temperature of the [illegible] vessel, and the temperature found to be 13·8°. [illegible] a temperature [illegible] was then poured in from a beaker until the calorimeter was two-thirds [illegible] after stirring well the temperature was 21·6°. A final weighing showed [illegible] 67·2 gms. of hot water had been added.

The heat given out by the hot water in cooling from 36·6° to 21·6° = 806·4 [illegible]

And the heat absorbed by the cold water = 81·7 × 9·2 = 751·6 [illegible]

And the heat absorbed by calorimeter = 806·4 − 751·6 = 54·8 [illegible]

∴ Heat absorbed by calorimeter = 54·8 cals. to raise its temperature 9·2°, and [illegible]

∴ The calorimeter requires 5·9 cals. [illegible] heat required to raise its temperature 1° = 5·9 cals.

The water equivalent is therefore 5·9 gms.

The third error mentioned above is reduced by hanging the calorimeter by three threads inside a larger vessel; this screens [illegible] from air currents, and, as will be understood later, lessens the [illegible] conduction. The radiation losses are smaller if both vessels [illegible] well-polished surfaces (p. 125). In addition it is arranged that [illegible] initial temperature of the calorimeter is slightly below and its [illegible] temperature nearly an equal amount above that of its surround[illegible] During the early stages of the experiment the calorimeter [illegible] receives heat from the room, but, as its temperature rises, it [illegible] out heat and the two may be made to balance approximately.

A convenient form of heater is shown in section in Fig. 18 [illegible] consists of two concentric brass tubes; in the inner one the sub[illegible]

to be heated is suspended in contact with the bulb of a thermometer, and the tube is closed by corks at each end to prevent air currents. Steam is made to circulate in the annular space between the tubes. The substance is thus heated without being wetted. It should be used in the form of thin sheet in order that it may acquire more quickly the temperature of the heater and of the cold water in the calorimeter. When the temperature of the hot body has remained steady for 15 mins. the temperature of the calorimeter is taken, the vessel is brought under the heater, the lower cork is removed, the upper one loosened, and the body is lowered rapidly into the water. The calorimeter is then removed some distance away, and, after stirring, the temperature of the mixture is noted. The specific heat is calculated from the equation already given. If the solid is in the form of a powder, or is soluble in water, it is enclosed in a copper case and the heat emitted by this is allowed for in the calculation.

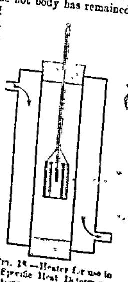


Fig. 18.—Heater for use in Specific Heat Determinations.

Specific Heat of Liquids.—If the specific heat, s, of the solid is known the same method may be used to find that of a liquid; the liquid in this case replaces the water in the calorimeter and it is s_1 which is calculated from the equation. When the liquid does not react chemically with water direct mixture may be used. A known weight of liquid is placed in the calorimeter and its temperature observed, water at a known temperature near 35° is added, and the temperature of the mixture is found. The calorimeter is then weighed to get the amount of water added and the calculation performed as in the previous case. Other methods which can be used for liquids are given on pp. 127 and [illegible].[1]

Specific Heats of Gases.—Consider a quantity of gas placed in a cylinder which is closed by a movable piston; the gas expands as its temperature is raised and pushes back the piston against the

[1] Barton and Black, "Practical Physics," pp. 63-64.

atmospheric pressure, *i.e.* it does work. It would ... h back the surrounding atmosphere just the same if the piston were removed. hence whenever a gas is heated at constant pressure it performs work. As will be seen in Chap. X it can do this only by using up some of the heat supplied to it, and this heat does not go to raise its temperature. It follows that when a gas is allowed to expand, more heat must be supplied to it to raise its temperature 1° than is necessary when its volume is kept constant, the excess is expended in the performance of work. In other words, we must consider two specific

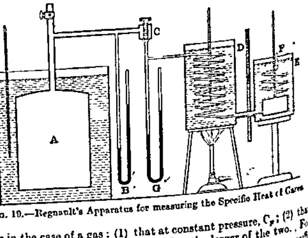


FIG. 19.—Regnault's Apparatus for measuring the Specific Heat of Gases

heats in the case of a gas: (1) that at constant pressure, C_p; (2) that at constant volume, C_v, the former being the larger of the two. For solids and liquids the expansions are so small that the two specific heats are practically equal.

Specific Heat of a Gas at Constant Pressure.—As in so many other cases the classical experiments on this subject are those of Regnault, his apparatus is shown, partly in section, in Fig. 19. The gas to be used was compressed in a reservoir, A, which was kept at a constant temperature in a tank of water. Experiments were first made to determine how the quantity of gas in the reservoir varied with the ..., so that from any future reading of the manometer, B, the ... the contained gas would be known. From A the gas flowed through a regulating cock, C, the heater D, the

calorimeter E, finally escaping into the atmosphere at F. The gas flow was kept steady by altering the cock C so that the pressure indicated by the manometer G was constant. The heater consisted of a long spiral of fine copper tubing immersed in a liquid maintained at a steady, high, temperature. The gas thus entered the calorimeter at a known temperature T. Here it passed through another copper spiral immersed in water and was cooled to the temperature of the calorimeter. Let t_1 and t_2 be the initial and final temperatures of the calorimeter, then the first portion of the gas was cooled from T° to t_1° and the final portion from T° to t_2°; the average temperature of the calorimeter during the passage of the gas was therefore $\frac{t_1+t_2}{2}$, and the heat lost by the gas was $m\left(T-\frac{t_1+t_2}{2}\right)s$, if m is its mass and s its specific heat. The heat gained by the calorimeter and its contents was calculated in the usual way and hence s was found. Owing to the time the experiment lasted the losses by conduction and radiation were large; an error in their determination appears to have caused a 2 per cent. inaccuracy in the final result. (See also p. 406.)

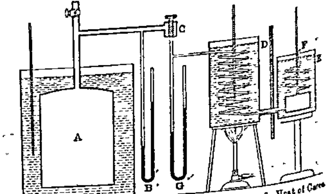
Table of Specific Heats.

Air (C_p)	0·2417	Ice	0·502
Air (C_v)	0·1715	Iron	0·119
Aniline	0·514	Lead	0·032
Bismuth	0·0304	Mercury	0·033
Copper	0·094	Turpentine	0·43
Glass	0·19	Zinc	0·093
Hydrogen	3·402		

Applications.—Calorimetry has some important scientific and technical applications other than the determination of specific heats. For example, it is important to the chemist to know how much heat is absorbed or evolved when chemical changes take place, and for the engineer it is necessary to know how much heat is evolved by burning a known weight of different kinds of fuel. These processes are made to take place in special kinds of calorimeter, where the heat evolved may be measured as in the preceding pages.

Dulong and Petit's Law.—Dulong and Petit, from their investigations of the specific heats of various chemical elements, were able to

atmospheric pressure, *i.e.* it does work. It would push back surrounding atmosphere just the same if the piston were removed, hence whenever a gas is heated at constant pressure it performs work. As will be seen in Chap. X it can do this only by using up some of the heat supplied to it, and this heat does not go to raise its temperature. It follows that when a gas is allowed to expand, more heat must be supplied to it to raise its temperature 1° than is necessary when its volume is kept constant, the excess is expended in the performance of work. In other words, we must consider two specific



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Dulong and
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deduce the law that the product of the specific heat and the atomic weight is constant. Regnault found that the law was approximately true if the substances were in the solid state; the mean value of the product (atomic weight × specific heat) is 6·2. Since the specific heat of a substance is found to vary with the temperature it is clear that the law cannot be universally true; in fact, recent experiments show that the specific heats of many substances are very much smaller at −250° C. than they are at the temperature of the laboratory.

EXAMPLES ON CHAPTER III

1. A mass of 200 gms. of copper, whose specific heat is 0·095, is heated to 100° and placed in 100 gms. of alcohol at 8° contained in a copper vessel whose mass is 25 gms. and the temperature rises to 28·5°. Find the specific heat of alcohol. (L. '89)

2. A copper vessel contains 100 gms. of water at 12°. When 56 gms. of water at 30° are added the resulting temperature of the mixture is 18°. What is the water equivalent of the vessel? A calorimeter with water equivalent 12 contains 100 gms. of water at 12°. When 100 gms. of metal at 100° are added the resulting temperature of the mixture is 20°. Find the specific heat of the metal. (L. '93)

3. Why is it difficult to measure the specific heat of a gas by the method of mixtures? What weight of gas of specific heat 0·25 entering at 100° would require to pass through an apparatus of which the heat capacity was 50 cals per degree before raising the temperature from 15° to 17°? (L. '10)

4. Eighty gms. of water at 35° are poured into a calorimeter containing 120 gms. of turpentine whose temperature is 15°. The calorimeter weighs 70 gms and its specific heat is 0·1. The specific heat of turpentine is 0·45; find the temperature of the mixture.

CHAPTER IV

LINEAR EXPANSION

Coefficient of Expansion.—It has already been shown that an increase in the temperature of a body is frequently accompanied by expansion; there are, however, exceptions to this rule. Below $-80°$ a rod of silica decreases in length when heated, and silver iodide contracts in volume up to a temperature of $142°$. In t[he] case of solids we may have to consider changes in length, area, [and] volume; with fluids we are concerned with volume changes alon[e]. This arises from the fact that a fluid takes the shape of the co[n]taining vessel and an increase in one dimension depends upon ho[w] much it is allowed to alter in the other two. The linear expansio[n] of a liquid is therefore an indefinite quantity, but its volume i[s] independent of the shape of the vessel and is perfectly definite at [a] given temperature.

When a bar is heated experiment shows that the increase i[n] length is proportional to the original length and to the temperatur[e] change, provided the latter is not too large. Suppose we have a ba[r] whose length at 0° is L_0 cms., and that when heated to $t°$ its lengt[h] becomes L_1, then each cm. has expanded $(L_1-L_0)/L_0$, and for 1° the expansion is $(L_1-L_0)/L_0t$. The ratio of the increase in length for 1° rise in temperature to the length at 0° is called the **coefficient of linear expansion**. Denoting this by l,

$$l=\frac{L_1-L_0}{L_0t}$$

or

$$L_1=L_0(1+lt)$$

When the temperature decreases, t must be put negative in this formula. If it is desired to compare the relative expansibilities of different solids we have merely to compare their coefficients of linear expansion. These coefficients are very small quantities, *e.g.* a bar

[illegible]

[illegible]

$$\frac{1}{1+x} = 1 - x + x^2 - \text{etc.}$$

$$\frac{1}{1-x} = 1 + x + x^2 + \text{etc.}$$

also $\quad (1+x)(1+y) = 1 + x + y + xy$

Suppose now $x = $ [illegible] and $y = $ [illegible], or quantities of the same order of magnitude as l above, then x^2, y^2, x^3, y^3, xy etc., are very small quantities indeed and we may neglect them in comparison with x and y

$$\frac{1}{1+x} = 1 - x$$

$$(1+x)(1+y) = 1 + x + y$$

and $\quad \dfrac{1}{(1+x)(1+y)} = \dfrac{1}{1+x} \cdot \dfrac{1}{1+y} = (1-x)(1-y) = 1 - x - y$

As an example, let the lengths of a bar at 0°, t_1° and t_2° be L_0, L_1 and L_2 respectively, and l its coefficient of linear expansion.

Then
$$L_1 = L_0(1 + lt_1)$$
$$L_2 = L_0(1 + lt_2)$$
$$\therefore \frac{L_2}{L_1} = \frac{L_0(1 + lt_2)}{L_0(1 + lt_1)} = \frac{1 + lt_2}{1 + lt_1}$$

accurately, or, using the above approximations, when lt_2 and lt_1 are small quantities,

$$\frac{L_2}{L_1} = (1 + lt_2) \cdot \frac{1}{1 + lt_1} = (1 + lt_2)(1 - lt_1) = 1 + l(t_2 - t_1)$$

Suppose we require the ratio of the length at 100° to that at 20° of a bar whose coefficient of linear expansion is 0·00002. The accurate formula gives 1·001599, the approximate formula 1·001600, practically the same result. From the last equation we get

$$l = \frac{L_2 - L_1}{L_1(t_2 - t_1)}$$

showing that, with these approximations, we need not refer the original length to 0° in order to calculate l; all that is required are the lengths at two known temperatures. How these are found is explained in the next paragraph.

Measurement of Coefficient of Linear Expansion.—A simple apparatus is shown in Fig. 20; it will also illustrate what errors are likely to arise in such an experiment. The rod AB to be experimented upon passes through corks up the centre of a wider tube C. Near the end A it is clamped between two metal knife-edges projecting slightly from wooden blocks fixed to a base-board. The end B, which is flat, can be made to touch a screw D of known pitch, say 0·5 mm. The large circular head of this screw is divided into 100 equal divisions; if it is turned through one division it will advance 1/200 mm. Steam is passed through C by the side tubes E, F, and a

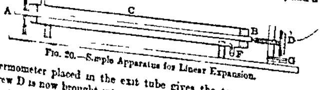

FIG. 20.—Simple Apparatus for Linear Expansion.

thermometer placed in the exit tube gives the temperature. The screw D is now brought into contact with B and the reading at the pointer G is taken. Cold water is next passed through tube C, causing the rod to contract. The contraction can be measured by noting how far the screw D has to be turned to bring it again in contact. The temperature of the water is noted, and finally the length of the rod from the knife-edges to B is measured by a scale; l can then be calculated from the last formula.

EXAMPLE.—For a glass rod 80 cms. long the screw had to be turned through 110 divisions on the circular head; the temperature of the steam was 100° and that of the water 15°. Hence $L_2 - L_1 = 110 \times 0{\cdot}005 = 0{\cdot}55$ mm. and $l = \dfrac{0{\cdot}055}{80 \times 85} = 0{\cdot}0000081$

There are several sources of error in this experiment: (1) Part of the rod is exposed to the outside air and will probably not reach the proper temperature; (2) The screw may become heated and so alter in length, this is minimised by having it in contact only when the reading is being taken, especially at the higher temperature; (3) The base board may expand and alter the position of the screw.

Lost time in the screw is avoided since it is always turned in the same direction. These errors are eliminated in the method now to be described, which, in principle, is that used at the International Bureau of Weights and Measures.[1]

Comparator Method.—The experimental bar (Fig. 21) is placed on rollers in one compartment of a metal trough, which is divided throughout the greater part of its length by a vertical division; in the other compartment a screw, worked by a motor, keeps water circulating past the bar. The temperature is read by two or more thermometers placed horizontally in the liquid. A second similar

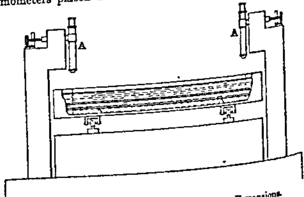

Fig. 21.—Comparator Method of measuring Expansions.

trough contains a standard bar having fine marks near the ends exactly one metre apart. Marks separated by approximately the same distance are also made on the experimental bar. There are, in addition, arrangements for levelling and for giving slight lateral or longitudinal displacements to either bar. The troughs can be run to and fro on a small tramway so that either rod can be brought beneath two vertical microscopes A, A, which are supported independently of the rest of the apparatus. Each microscope carries cross-wires in the eye-piece which can be moved by a micrometer screw with divided head, similar to the one already described. The movement of the marks as small as 0·001 mm. can be measured. The troughs having been filled with ice-cold water, the standard

[1] For a simple modification, see Barton and Black, "Practical Physics," p

s placed beneath the microscopes and the cross-wires are adjusted so that they appear to coincide with the marks when seen through the instruments. The second trough is now brought into position and the cross wires adjusted as before by moving the micrometer screws. The amount of this movement shows at once by how much the distance between the marks on the experimental bar differs from one metre and hence the length at 0° is found. The water is next heated to a known temperature and the resulting expansion of the bar is measured by the cross-wires and screws. The standard bar, still at 0°, is finally brought under again to ensure that the distance apart of the

FIG. 22.—To show Stress caused by Cooling.

microscopes has remained unaltered. The length being known at two temperatures the coefficient of linear expansion can be calculated.

TABLE I

Coefficients of Linear Expansion.

Brass	0·0000189	Nickel steel (10% nickel)	0·0000082
Copper	0·0000172	Platinum	0·0000081
Glass (tube)	0·0000081	Porcelain	0·0000083
Iron (soft)	0·0000122	Fused silica	0·00000059
Invar (Nickel steel, 36% nickel)	0·0000017	Steel (untempered)	0·0000108
		Zinc	0·0000294

(The above numbers represent average values.)

Applications.—

as tightly as possible; when it cools the stress is so great that th[e] small bar, B, which passes through a hole in the end of A, is broke[n].

The time of swing of a pendulum depends upon its length; therefore, the temperature changes, the rate of a clock will be alter[ed] unless we can arrange to keep the pendulum bob at a fixed distance from the point of suspension. This is done in various ways. Fig. 23 shows a form of Harrison's gridiron pendulum. The middle and two outer rods are made of iron, the remaining pair of zinc. The middle rod passes freely through the cross-piece B. Expansion of the iron alone will evidently lower the bob, expansion of the zinc alone will raise it, since Z can only expand upwards. When the temperature of the whole alters these changes may be made to balance each other. Denoting the lengths of the rods by the letters on them in the figure, the distance of the bob from $A = I - Z + S$.

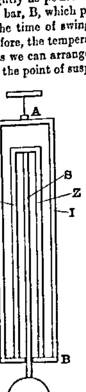

FIG. 23.—Gridiron Pendulum.

When the temperature rises by $t°$ this becomes

$$(I+S)(1+0{\cdot}0000122t) - Z(1+0{\cdot}0000291t)$$
from Table I

This must be equal to the original length if the time of vibration is to remain unaltered. Hence

$$(I+S)(1+0{\cdot}0000122t) - Z(1+0{\cdot}0000291t) = I - Z + S$$

whence we find readily

$$\frac{I+S}{Z} = \frac{291}{122}$$

showing that the ratio of the lengths of the iron and zinc rods is inversely as their coefficients of linear expansion.

The rate of a watch is regulated by the elasticity of the spring and the size and mass of the balance wheel. A rise in temperature lessens the elasticity and increases the size of the wheel, each of which causes the watch to lose time, though it is the former wh[ich]

...lly effective. This is counterbalanced by causing the expans... bring the weight of the wheel rim nearer the centre.

EXPERIMENT.—Rivet a strip of zinc 20 cms. long and 1 cm. wide to a sim... p of iron to form a compound strip of the same length and heat it in a fl... becomes curved with the zinc on the outside. This is because zinc exp... re than iron and the metals can only take up their appropriate length... rving in this manner.

The same principle is applied to the balance wheel of a w... ig. 24). The rim is made of a compound strip of two or ... etals with the more expansible metal utside; as the temperature rises the trip curves inwards, and compensation may be attained by properly distributing the weight of the rim.

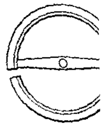

FIG. 24.—Balance W... of a Watch.

It will be noticed from Table I. that invar has a very small coefficient of expansion; it should, therefore, prove useful for clock pendulums, standards of length, etc. If a piece of iron wire is sealed through a glass tube, the joint usually fractures as it cools owing to the equal contraction of the two substances. Platinum and ...eel (47 per cent. nickel) have, as shown in Table I., about the ...pansion as glass; hence they may be used more safely f... urpose, e.g. in the construction of incandescent electric lamp...

Thick-bottomed drinking-glasses frequently crack if hot ... poured into them owing to the unequal expansion of the inn... outer layers. Glass is a bad conductor of heat (p. 115) ... hinders the rapid equalisation of temperature in the different p... Fused quartz or silica has a very small expansion; vessels ... this substance may be plunged into a very hot bath without breakage.

EXAMPLES ON CHAPTER IV

1. A base line [illegible] miles long is to be laid out by means of an iron ch... [illegible] is known at 0°. Find the percentage error caused by negle... [illegible] of the iron if the coefficient of linear expansion is 0·00001... average temperature is 17°.

2. A rod is found to be 100 cms. long at [illegible]° and 100·1 cms. lon...

as [illegible] as possible; when it cools the spring is [illegible] great [illegible] [illegible] fan, B, which passes through a hole in the end of A, is [illegible]

The time of swing of a pendulum depends upon its length; [illegible] [illegible] as the temperature changes, the rate of a clock will be [illegible] [illegible] we can arrange to keep the pendulum bob at a fixed [illegible] from the point of suspension. This is done in various ways. Fig. [illegible] shows a form of Harrison's gridiron pendulum. The middle and two outer rods are made [illegible] iron, the remaining pair of zinc. The [illegible] rod passes freely through the cross-piece [illegible] Expansion of the iron alone will evidently lower the bob, expansion of the zinc alone will raise it, since Z can only expand upward. When the temperature of the whole [illegible] these changes may be made to balance [illegible] other. Denoting the lengths of the rods [illegible] the letters on them in the figure, the distance of the bob from A $= I - Z + S$.

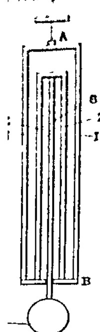

FIG. 23.—Gridiron Pendulum.

When the temperature rises by $t°$ [illegible] becomes

$$(I+S)(1+0{\cdot}0000122t)-Z(1+0{\cdot}0000[illegible]$$

from Table [illegible]

This must be equal to the original length [illegible] the time of vibration is to remain unaltered. Hence

$$(I+S)(1+0{\cdot}0000122t)-Z(1+0{\cdot}0000294t) = I-Z+[illegible]$$

whence we find readily

$$\frac{I+S}{Z}=\frac{294}{122}$$

showing that the ratio of the lengths of [illegible] iron and zinc rods is inversely as their coefficients of expansion.

The rate of a watch is regulated by the elasticity of the spring and the size and mass of the balance wheel. A rise in temperature decreases the elasticity and increases the size of the wheel, each of which causes the watch to lose time, though it is the former which

ly effective. This is counterbalanced by causing the expansion ring the weight of the wheel rim nearer the centre.

:XPERIMENT.—Rivet a strip of zinc 20 cms. long and 1 cm. wide to a similar of iron to form a compound strip of the same length and heat it in a flame comes curved with the zinc on the outside. This is because zinc expands than iron and the metals can only take up their appropriate lengths by ing in this manner.

The same principle is applied to the balance wheel of a watch g. 24). The rim is made of a compound strip of two or more tals with the more expansible metal tside; as the temperature rises the ip curves inwards, and compensation ay be attained by properly distributing e weight of the rim.

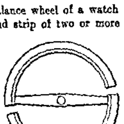

FIG. 24.—Balance Wheel of a Watch.

It will be noticed from Table I. that var has a very small coefficient of xpansion; it should, therefore, prove seful for clock pendulums, standards of ength, etc. If a piece of iron wire is ealed through a glass tube, the joint usually fractures as it cools owing to the unequal contraction of the two substances. Platinum and nickel steel (45 per cent. nickel) have, as shown in Table I., about the same expansion as glass; hence they may be used more safely for th purpose, e.g. in the construction of incandescent electric lamps.

Thick-bottomed drinking-glasses frequently crack if hot wate is poured into them owing to the unequal expansion of the inner an outer layers. Glass is a bad conductor of heat (p. 115) and ders the rapid equalisation of temperature in the different portion ed quartz or silica has a very small expansion; vessels made substance may be plunged into a very hot bath without fear akage.

EXAMPLES ON CHAPTER IV

1. A base line [illegible] is to be laid out by means of an iron chain wh gth is known at 0°. Find the percentage error caused by neglecting pansion of the iron if the coefficient of linear expansion is 0·000012 and rage temperature is 15°.

2. A rod is found to be 100 cms. long at 50° and 100·1 cms. long at 1[illegible]

as tightly as possible; when it cools the stress is so great
small bar, B, which passes through a hole in the

The time of swing of a pendul
therefore, the temperature c
unless we can arrange to ke
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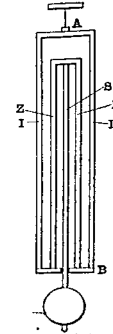

FIG. 23.—Gridiron Pendulum.

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CUBICAL EXPANSION

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e temperature change and to the original volume of the body
increase in volume when the temperature is raised 1° divided
e volume at 0° is called the coefficient of cubical expansion.
oting this by c, if V_0 = vol. at 0°, V = vol. at t°,

$$c = \frac{V - V_0}{V_0 t}$$

$$V = V_0(1 + ct)$$

n the case of linear expansion, on account of the smallness of l,
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tion, is the coefficient of cubical expansion c, hence $c = 3l$.
olid homogeneous bodies the coefficient of cubical expansion is
times the coefficient of linear expansion.
ppose we have a spherical glass flask filled with a solid core of

Assuming that it expands [illegible] ... its coefficient of cubical expansion [illegible]
at any temperature and calculate its coefficient of cubical expansion.

3. If it takes a force of 20,000 kilos./cm.^{2} to produce a 1 per cent. diminution of length in an iron bar, what force would you expect it to require to prevent a bar 8 cms. long, 3 cms. wide, and 2 cms. deep, from expanding lengthways when raised 500°? Coefficient of expansion of iron = 0·0000122. (L. '04.)

4. The height of the barometer at 18° is found to be 76 cms. when measured with a brass scale which is correct at 0°. Find the actual length of the mercury column. Coefficient of expansion of brass is 0·0000182.

CHAPTER V

CUBICAL EXPAN

Volume Expansion of Solids and Liquids.—When a homogeneous body is heated it is found, as the result of experiment, that the increase in volume due to a small rise of temperature is proportional to the temperature change and to the original volume of the body. **The increase in volume when the temperature is raised 1° divided by the volume at 0° is called the coefficient of cubical expansion.** Denoting this by c, if V_0 = vol. at 0°, V = vol. at $t°$,

$$c = \frac{V - V_0}{V_0 t}$$

or

$$V = V_0(1 + ct)$$

In the case of linear expansion, on account of the smallness of l we saw that it did not introduce serious error if the original length was measured at some temperature other than 0°. The coefficient of cubical expansion is a much larger quantity, especially in the case of liquids and gases; it is better, therefore, to refer our definition to the volume at 0°, although the difference will not be large in the case of liquids and will be still smaller for solids.

Consider a cube of a solid body 1 cm. in side at 0°, and let the coefficients of linear and cubical expansion of the material be l and c respectively. If the temperature be raised 1° each side becomes $(1 + l)$, and the volume becomes $(1 + l)^3 = 1 + 3l + 3l^2 + l^3$. From what has been said about small quantities we may take this as being $(1 + 3l)$, and the increase in volume is thus $3l$. But this, from definition, is the coefficient of cubical expansion c, hence $c = 3l$. **For solid homogeneous bodies the coefficient of cubical expansion is three times the coefficient of linear expansion.**

Suppose we have a spherical glass flask filled with a solid core of

may also get the true volume of the liquid by multiplying its apparent volume by $(1+g)$.

$\therefore$ true volume of the liquid $V_0(1+c) = V_0(1+c_a)(1+g)$

or
$$1+c = 1+c_a+g+c_a g$$

The coefficients being small the term $c_a g$ may be neglected and

$$c = c_a + g$$

To get the true coefficient of expansion of a liquid we must add the apparent coefficient to the coefficient of cubical expansion of the vessel

The apparent coefficient can readily be observed in the dilatometer, Fig 25. This consists of a glass bulb, whose volume is known at 0° C., attached to a stem graduated in c.cms. Liquid is placed in the apparatus and ts apparent volume at two different temperatures is noted by the graduations, the apparent coefficient of xpansion can then be calculated. If we add to this pparent coefficient three times the linear coefficient for lass we should expect to get the true coefficient for the quid. This procedure, however, is faulty, since glass usually far from homogeneous, and after determining for a glass tube a bulb must be blown on it, which ight greatly alter its expansibility. The method is erefore inaccurate. If c could be determined for some quid independently of the envelope, we could afterwards serve c_a for the same liquid in the dilatometer, and, ing the above equation, calculate g. The dilatometer uld then be used to find c_a for any other liquid, and nce c.[1] We proceed to show how the true (or absolute) pansion of a liquid is found.

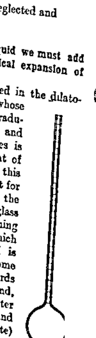

FIG. 25.—Dilatometer

Absolute Expansion of Mercury.—The apparatus own in Fig. 26 is a simplified form of that used by long and Petit. BAA'B' represents a glass tube containing rcury which is at different temperatures in the upright limbs; ' is horizontal. From a well-known hydrostatical principle, the ensity of pressure at A must equal that at A' when there is ilibrium, independently of whether the tubes have equal

[1] Barton and Black, "Practical Physics," p. 61.

e same material; when heated they ... solid body. The flask would expand by an equal amount if the ... were absent. It is seen from this that hollow bodies increase in volume by the same amount as solid bodies of the same dimensions. Thus a glass tube and a glass rod of the same diameter at the same temperature will still have equal diameters if the temperature of each is changed by the same amount.

Effect of Temperature on Density.—The density of a body is defined as the mass of unit volume. If d_0 is the density of a body at 0° when the volume is V_0, the mass is V_0d_0. At another temperature, $t°$, let the volume be V and the density d. The mass being constant,

$$V_0d_0 = Vd$$

or

$$\frac{d}{d_0} = \frac{V_0}{V}$$

But

$$V = V_0(1 + ct)$$

$$\therefore \frac{d}{d_0} = \frac{V_0}{V_0(1 + ct)} = \frac{1}{1 + ct}$$

This result is important in dealing with the expansion of fluids; it shows that we can calculate the coefficient of cubical expansion if we can compare the densities at 0° and $t°$.

Apparent and True Coefficient of Expansion of a Fluid.—We have already seen, p. 19, that the expansion of a liquid is partly masked by that of the vessel containing it. The expansion observed under such conditions is called the apparent expansion. Similarly the coefficient of apparent expansion of a fluid, which we will denote by c_a, is the apparent increase in volume for 1° rise of temperature divided by the volume at 0°. The true coefficient c will be greater than this since it takes into account the increase in volume of the vessel. Suppose we have a glass bulb, whose volume at 0° is V_0 c.cms., surmounted by a stem graduated in c.cms., and let the coefficient of cubical expansion of the glass be g. Let the bulb be filled at 0° with a liquid whose true coefficient is c. At 1° the true volume of the liquid is $V_0(1 + c)$, but owing to the expansion of the glass the graduations are incorrect and the volume as read on the stem will be less than this; it will apparently be $V_0(1 + c_a)$. At this temperature each c.cm. of the vessel has expanded to $(1 + g)$, so that ...

Hence if the heights of two balancing columns are measured when they are at different temperatures, c can be found without any knowledge of the expansion of glass.

To carry out the measurements in a form suitable for a simple laboratory experiment the columns are surrounded by wider tubes as in the figure. Through these a stream of ice-cold water is first run, and the quantity of mercury is adjusted until the ends of the columns are just visible above the corks B, B'. The heights of the surfaces above the axis of AA' are measured, corresponding with H_0 above. Steam is next passed round A' B'; the temperature in each case is given by thermometers projecting through the corks at B', B. The length of the warm column is again measured, giving $H - H_0$, and c is calculated as above.

The apparatus in this simple form has several disadvantages: (1) It does not give a continuous series of temperatures in the hot limb; this can be obviated by having the column A'B' surrounded by an oil bath, which must be well stirred; (2) The thermometer may not give the mean temperature of the hot column; (3) The surfaces are exposed and may change in temperature while the observations are being made.

Fig. 27 shows the principle of an apparatus used by Regnault, and more recently improved by Callendar, to overcome these defects.

The vertical tubes AB, A'B', from one to two metres long and a cm. in diameter, are bent twice at right angles so that the portions BC, B'C' are horizontal. AA' is made of narrow bore to prevent the circulation of currents of mercury from one vertical tube to the other. Water cooled to 0° by ice in M is steadily passed through the wide tube surrounding AB; we will suppose also that it drips on blotting paper wrapped round CD and C'D'. A'B' is surrounded by oil, which is first heated by passing an electric current through the wire coil Q and is then forced past the mercury column in the direction of the arrows by a small centrifugal pump R. The mean temperatures of the long columns are given by platinum thermometers P' and P (p. 389), whose bulbs extend the whole lengths of AB, A'B'; that of the short tubes CD, C'D' by mercury thermometers placed in contact with them. The heights of the various columns are measured by a cathetometer. This consists of a horizontal telescope, having cross-wires in the eye-piece, which moves up and down a vertical graduated bar. The telescope is first focussed

diameters or not. If H_0 and H are the lengths of the columns in AB and A'B' respectively, d_0 and d the corresponding densities, then

$$Hd = H_0 d_0$$ from p. 43,

or

$$\frac{H}{H_0} = \frac{d_0}{d} = 1 + ct$$

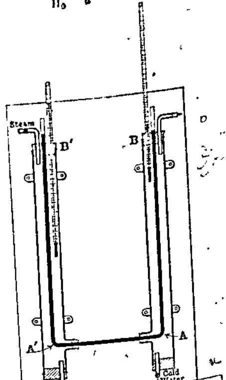

FIG. 26.—Dulong and Petit's Apparatus.

if the temperatures of the limbs are 0° and $t°$ and c is the
of expansion of the liquid;

$$\therefore c = \frac{H - H_0}{H_0 t}$$

Hence if the heights of two balancing columns are measured when they are at different temperatures, c can be found without any knowledge of the expansion of glass.

To carry out the measurements in a form suitable for a simple laboratory experiment the columns are surrounded by wider tubes as in the figure. Through these a stream of ice-cold water is first run, and the quantity of mercury is adjusted until the ends of the columns are just visible above the corks B, B′. The heights of the surfaces above the axis of AA′ are measured, corresponding with H_0 above. Steam is next passed round A′ B′; the temperature in each case is given by thermometers projecting through the corks at B′, B. The length of the warm column is again measured, giving $H - H_0$, and c is calculated as above.

The apparatus in this simple form has several disadvantages: (1) It does not give a continuous series of temperatures in the hot limb, this can be obviated by having the column A′B′ surrounded by an oil bath, which must be well stirred; (2) The thermometer may not give the mean temperature of the hot column; (3) The surfaces are exposed and may change in temperature while the observations are being made.

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[illegible]

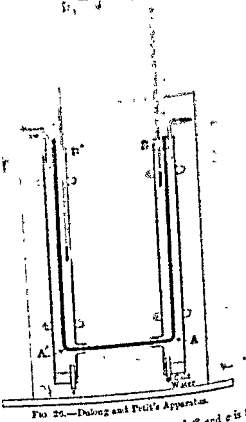

FIG. 25.—Dulong and Petit's Apparatus.

if the temperatures of the limbs are 0° and t° and c is the coefficient of expansion of the liquid;

$$\therefore c = \frac{H - H_0}{H_0 t}$$

diameters or not. If H_0 and H are the lengths of the columns in AB and A'B' respectively, d_0 and d the corresponding densities, then

$$Hd = H_0 d_0 \quad \text{from p. 48}$$

or

$$\frac{H}{H_0} = \frac{d_0}{d} = 1 + ct$$

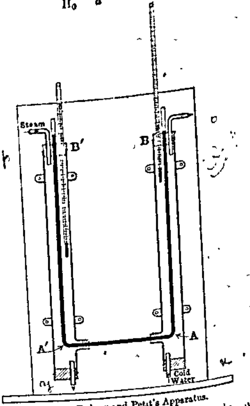

FIG. 26.—Dulong and Petit's Apparatus.

f the temperatures of the limbs are 0° and t° and c is the of expansion of the liquid ;

$$\therefore c = \frac{H - H_0}{H_0 t}$$

or

$$\frac{H}{H_0+h-h'}=\frac{d_0}{d}=1+ct \quad \text{(p. 48)}$$

whence

$$c=\frac{H-H_0-h+h'}{(H_0+h-h')t}$$

Between 0° and 100° c is found to be 0 000182, as the temperature i raised the coefficient increases.

Methods of determining the Apparent Expansion of Liquids.—1) *The Dilatometer method* already mentioned is chiefly used when volatile liquids are concerned, as there is little opportunity for evaporation owing to the small surface exposed.

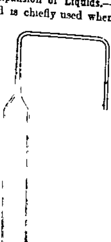

Fig 28.—Weight Thermometer.

(2) *Weight thermometer.* The glass apparatus shown in Fig. 28 has a reservoir about cms. long which is continued by a capillary tube bent twice at right angles. It is cleaned, dried, and weighed, and is then supported by copper gauze with the end of the tube under the surface of the liquid. It is filled by alternate heating and cooling, and during the final cooling is placed in melting ice; it is thus completely filled with liquid at 0°. It is next transferred to a bath which can be heated to any suitable temperature $t°$; on account of expansion some of the liquid overflows and is received in a weighed beaker. A further weighing of the beaker and its contents gives the mass that has overflowed. The bulb and remaining liquid are also weighed, subtracting the weight of the glass we have the mass of liquid left in. Let m be the mass that overflows, M the mass left in, and d_0 the density of the liquid at 0°. Since we are finding the apparent expansion the increase in volume of the glass is neglected. The mass of liquid filling the thermometer at 0° is $(M+m)$.

∴ volume of thermometer $= (M+m)/d_0$

That a mass M of liquid, whose volume at 0° is M/d_0, fills the thermometer at $t°$ when the volume is ...

so that the cross-wires appear to be in line with the axis of AA'; it is then moved so that they appear to coincide with the axis of BB'. The distance it has been displaced gives the vertical length AB, and similarly for the other columns. Let h', H, h and H_0 be the

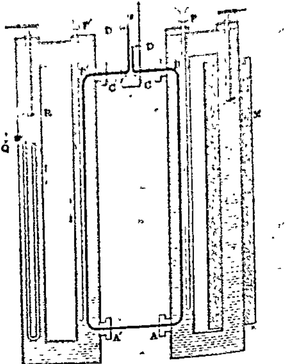

FIG. 27.—Callendar's Apparatus.

lengths of C'D', A'B', CD and AB respectively, when A'B' has temperature $t°$ and the others are at 0°. If d and d_0 are the densities of mercury at these temperatures the pressure at A' is $(h'd_0 + H$ that at A is $(hd_0 + H_0d_0)$.

$$\therefore h'd_0 + Hd = hd_0 + H_0d_0$$

$$Hd = (H_0 + h - h')d_0$$

letermined by the dilatometer method. It is then partially fille with lead shot to make it sink, the neck is sealed, and the whol lung from the arm of a balance by a thin wire and immersed in th iquid. In these methods it would be better to replace the glass by quartz since its expansion is more definite.[1]

Expansion of Water.—If a dilatometer containing water at 0° i gradually heated the liquid is seen to contract until a temperature lear 4° is reached, after which it continually expands, showing that vater has a maximum density near 4°. The exact temperature bserved will depend on the expansion of the glass. Joule's method gives the temperature of maximum density directly. Two cylin-lrical vessels, A, B (Fig. 29), contain-ng water, communicate below through tube which can be closed by a tap, bove through an open trough. Their emperatures are adjusted so that one s below and the other above 4°. If he density of the liquid is greater in A han in B, when the tap is opened a urrent sets in in the direction ACDBA, ecause the pressures at C and D are nequal. This is rendered evident by small bead floating in the trough. 'wo temperatures are found at which rater has the same density; these are ltered until they are nearly equal and heir mean taken as the temperature f maximum density.

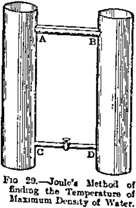

FIG 29.—Joule's Method of finding the Temperature of Maximum Density of Water.

This singular behaviour of water has an important influence on nimal and vegetable life. During a frost the surface layers of a ond are first cooled, they increase in density and sink, and are eplaced by the warmer layers from below. This proceeds until the hole is reduced to 4°, when any further cooling produces a decrease 1 density. The water which is cooled below 4° consequently floats n the surface and is finally frozen, while the lower portions have a niform temperature of 4°, thus protecting plants and animals from eing frozen.

‌viding by the volume of the liquid at 0°, M/d_0, and by the ‌mperature change $t°$, we get the coefficient of apparent expansion c_a

$$\therefore c_a = \frac{\frac{m}{d_0}}{\frac{M \cdot t}{d_0}} = \frac{m}{Mt} .$$

The method is useless for volatile liquids, since obviously a large ‌roportion of the mass overflowing will evaporate.

The apparatus may be replaced by a small flask with a narrow ‌eck, or a specific gravity bottle, and the liquid adjusted to a fixed ‌ark upon it by means of a pipette. If mercury is the liquid used, ‌nce c is known from the last paragraph, we can calculate the cubical ‌xpansion of the glass from $c = c_a + g$. The apparatus can then be ‌sed to find c_a for any other liquid, and as g is known the true ‌oefficient can be found.

(3) *Hydrostatic method.* Let a solid be weighed (1) in air, (2) ‌ompletely immersed in a liquid at 0°, (3) in the same liquid at $t°$. ‌et the loss of weight at 0° be m_0 and at $t°$ be m; these represent ‌he masses of liquid having the same volume as the solid at 0° and $t°$ ‌espectively. The volume of liquid displaced at 0° is m_0/d_0, that at ‌ is m/d, where d_0 and d are the corresponding densities of the liquid. ‌ence the volume of the solid at 0° is m_0/d_0, and at $t°$ this has ex‌anded to $m_0(1 + gt)/d_0$, g being the coefficient of cubical expansion ‌f the solid. (See equation p. 47.)

Equating the volume of the solid at $t°$ to that of the displaced ‌iquid at the same temperature,

$$\frac{m}{d} = \frac{m_0}{d_0}(1 + gt)$$

$$\therefore \frac{m}{m_0} = \frac{d}{d_0}(1 + gt)$$

‌nd
$$\frac{m}{m_0} = \frac{1 + gt}{1 + ct}$$

‌ince
$$\frac{d}{d_0} = \frac{1}{1 + ct}$$

‌ = coeff. of expansion of the liquid.

‌ therefore find either c or g, provided the other is known ‌d can be made in the form of a glass bulb and its coefficient

letermined by the dilatometer method. It is then partiall with lead shot to make it sink, the neck is sealed, and the hung from the arm of a balance by a thin wire and immersed liquid. In these methods it would be better to replace the g quartz since its expansion is more definite.[1]

Expansion of Water.—If a dilatometer containing water gradually heated the liquid is seen to contract until a temp near 4° is reached, after which it continually expands, show water has a maximum density near 4°. The exact temp observed will depend on the expansion of the glass. Joule's gives the temperature of maximum density directly. Tw drical vessels, A, B (Fig. 29), containing water, communicate below through a tube which can be closed by a tap, above through an open trough. Their temperatures are adjusted so that one is below and the other above 4°. If the density of the liquid is greater in A than in B, when the tap is opened a current sets in in the direction ACDBA, because the pressures at C and D are unequal. This is rendered evident by a small bead floating in the trough. Two temperatures are found at which water has the same density; these are altered until they are nearly equal and their mean taken as the temperature of maximum density.

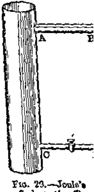

Fig. 29.—Joule's finding the Tem Maximum Densit

This singular behaviour of water has an important in animal and vegetable life. During a frost the surface pond are first cooled, they increase in density and sin replaced by the warmer layers from below. This procee whole is reduced to 4°, when any further cooling produce in density. The water which is cooled below 4° consequ on the surface and is finally frozen, while the lower port uniform temperature of 4°, thus protecting plants and a being frozen.

The currents set up in a cooling liquid are well show

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$$\therefore c_a = \frac{\frac{m}{d_0}}{\frac{M \cdot t}{d_0}} = \frac{m}{Mt}.$$

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The apparatus may be replaced by a small flask with a narrow neck, or a specific gravity bottle, and the liquid adjusted to a fixed mark upon it by means of a pipette. If mercury is the liquid used, since c is known from the last paragraph, we can calculate the cubical expansion of the glass from $c = c_a + g$. The apparatus can then be used to find c_a for any other liquid, and as g is known the true coefficient can be found.

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Equating the volume of the solid at $t°$ to that of the displaced liquid at the same temperature,

$$\frac{m}{d} = \frac{m_0}{d_0}(1 + gt)$$

$$\therefore \frac{m}{m_0} = \frac{d}{d_0}(1 + gt)$$

and

$$\frac{m}{m_0} = \frac{1 + gt}{1 + ct}$$

s-are

$$\frac{d}{d_0} = \frac{1}{1 + ct}$$

coeff. of expansion of the liquid.

... therefore find either c or g, provided the other is known ...

... can be made in the form of a glass bulb and its coefficient ...

determined by the dilatometer method. It is then partially filled with lead shot to make it sink, the neck is sealed, and the whole hung from the arm of a balance by a thin wire and immersed in the liquid. In these methods it would be better to replace the glass by quartz since its expansion is more definite.[1]

Expansion of Water.—If a dilatometer containing water at 0° is gradually heated the liquid is seen to contract until a temperature near 4° is reached, after which it continually expands, showing that water has a maximum density near 4°. The exact temperature observed will depend on the expansion of the glass. Joule's method gives the temperature of maximum density directly. Two cylindrical vessels, A, B (Fig. 29), containing water, communicate below through a tube which can be closed by a tap, above through an open trough. Their temperatures are adjusted so that one is below and the other above 4°. If the density of the liquid is greater in A than in B, when the tap is opened a current sets in in the direction ACDBA, because the pressures at C and D are unequal. This is rendered evident by a small bead floating in the trough. Two temperatures are found at which water has the same density; these are altered until they are nearly equal and their mean taken as the temperature of maximum density.

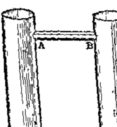


FIG. 29.—Joule's Method of finding the Temperature of Maximum Density of Water.

This singular behaviour of water has an important influence on animal and vegetable life. During a frost the surface layers of a pond are first cooled, they increase in density and sink, and are replaced by the warmer layers from below. This proceeds until the whole is reduced to 4°, when any further cooling produces a decrease in density. The water which is cooled below 4° consequently floats on the surface and is finally frozen, while the lower portions have a uniform temperature of 4°, thus protecting plants and animals from being frozen.

The currents set up in a cooling liquid are well shown in II

[1] See also Barton and Black, "Practical Physics," p. 55.

Then when the observed height H is 76 cms. the true height, re
to 0° c, is 75 81 cms.[1]

Exposed Stem Correction for a Thermometer.—With the no
of p. 25 it is seen that if the mercury occupying n divisions is l
from t_2 to t, so as to be at the same temperature as the rest
md, it will expand $n\sigma(t - t_2)$ divisions. This
the amount to be added to the observed
nperature t_1 to get the true temperature t,
$t = t_1 + n\sigma(t - t_2)$. The correction should
ver be large as it is not very trustworthy,
nce it will be sufficiently near the truth if
on the right side is replaced by the nearly
jual temperature t_1, and the formula becomes
$= t_1 + n\sigma(t_1 - t_2)$.

Applications.—The expansion of a liquid is
sed to regulate the supply of heat to a bath
hich it is desired to maintain at a constant
emperature. Fig. 31 shows one form of gas
egulator. The glass bulb A is filled with a
nighly expansible liquid like toluene, the lower
part of A and the narrow tube to B are filled
with mercury. The apparatus is placed in the
bath, which should be well stirred, and gas
from the main enters at D, travels in the path
shown by the arrows, and goes from E to the
burner underneath the bath. When a certain
temperature is reached the expansion of the
toluol causes the mercury to close the tube
B and cut off the gas. To save it from being
extinguished a small bye-pass is provided at C
which allows sufficient gas to pass to keep the flame alight
the temperature falls the toluol contracts and the full supp
again passes through B. The temperature can be kep
constant for days by this device.

FIG. 31 -
Regula

EXAMPLES ON CHAPTER V

1 Describe a method of measuring the coefficient of expansio
f rod. A solid at 0° when immersed in water displaces 300 cub. in

[1] For further correcti ... Watson and Black, "Practical Physi

apparatus, Fig. 30. Water at the room temperature is placed in the upright cylinder into which two thermometers project, and a freezing mixture of ice and salt is placed in the trough A. This causes the temperature of the lower thermometer to fall to 4°, without at first, seriously affecting the upper one. The reading of the lower thermometer then remains stationary and that of the upper falls rapidly until it reaches zero.

Correction of a Barometer for Temperature.—The observed height of a barometer will vary with the temperature on account of the expansion of the scale and of the mercury; it must therefore be reduced to a standard temperature of 0°. Let H, H_0 cms. be the observed heights at $t°$ and 0° respectively, d and d_0 the corresponding densities of mercury, l the coefficient of linear expansion of the scale, c the coefficient of cubical expansion of mercury. The graduations on the scale being supposed correct at 0°, the true length of the mercury column at $t°$ is $H(1+lt)$. The atmospheric pressure in grams per cm.^{2} is thus $H(1+lt)d$, which if it were measured by a barometer at 0° it would be H_0d_0

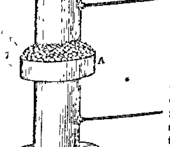


FIG. 30 —Hope's Apparatus.

$$\therefore H_0d_0 = H(1+lt)d$$

and

$$H_0 = H(1+lt)\frac{d}{d_0}$$

But

$$\frac{d}{d_0} = \frac{1}{1+ct}$$

$$\therefore H_0 = \frac{H(1+lt)}{1+ct}$$

or approximately

$$H_0 = H\{1 - t(c-l)\} \quad \text{(p. 40)}$$

To get some idea of the magnitude of this correction suppose the scale is brass, for which $l = 0{\cdot}000018$, the temperature 15°, and take the coefficient of cubical expansion of mercury to be 0·00018

en when the observed height H is 76 cms. the true height, reduced 0° c, is 75 81 cms[1]

Exposed Stem Correction for a Thermometer.—With the notation f p. 25 it is seen that if the mercury occupying n divisions is heated om t_2 to t, so as to be at the same temperature as the rest of the quid, it will expand $n\sigma(t - t_2)$ divisions. This . the amount to be added to the observed emperature t_1 to get the true temperature t, r $t = t_1 + n\sigma(t - t_2)$. The correction should ever be large as it is not very trustworthy, ence it will be sufficiently near the truth if . on the right side is replaced by the nearly equal temperature t_1, and the formula becomes $t = t_1 + n\sigma(t_1 - t_2)$.

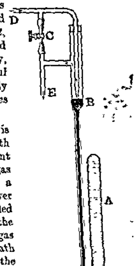


FIG. 31.—Gas Regulator.

Applications.—The expansion of a liquid is used to regulate the supply of heat to a bath which it is desired to maintain at a constant temperature. Fig. 31 shows one form of gas regulator. The glass bulb A is filled with a highly expansible liquid like toluene, the lower part of A and the narrow tube to B are filled with mercury. The apparatus is placed in the bath, which should be well stirred, and gas from the main enters at D, travels in the path shown by the arrows, and goes from E to the burner underneath the bath. When a certain temperature is reached the expansion of the toluol causes the mercury to close the tube B and cut off the gas. To save it from being extinguished a small bye-pass is provided at C which allows sufficient gas to pass to keep the flame alight. When the temperature falls the toluol contracts and the full supply of gas again passes through B. The temperature can be kept nearly constant for days by this device.

EXAMPLES ON CHAPTER V

1. Describe a method of measuring the coefficient of expansion of a metal rod. A solid at 0° when immersed in water displaces 500 cub. in.; at 30° it

[1] For further correct... son and Black, "Practical Physics," p. 68.

displaces 503 cub. in. Find its mean coefficient of linear expansion between 0° and 30°. (L. '80.)

2. Find the value in grams weight, and in dynes per cm.², of a pressure able to sustain a 60 cm. column of mercury at 0°. Find what pressure would be exerted by the same height of mercury at 100°, if its density at 0° be 13·6 and its mean coefficient of expansion be 0·00018. (L. '90.)

3. A glass bulb with a fine uniform stem weighs 10 gms. when empty, 117·3 gms. when the bulb only is filled with mercury, and 119·7 gms. when a length of 10·4 cms. of the stem is also filled with mercury. Calculate the relative coefficient of expansion for temperature of a liquid which, when placed in the same bulb, expands through the length from 10·4 to 12·9 cms. of the stem when warmed from 0° to 23°. The density of mercury is 13·6 gms. per cm.³ (L. '93.)

4. A mercury thermometer at 0° contains 2 c.c. of mercury and the distance between the fixed points is 30 cms. Calculate the diameter of the tube at 0°, given the coefficient of cubical expansion of mercury is 0·00018 and of glass 0·00003. (L. '00.)

5. A specific gravity bottle holds 50 gms. of water at 4°. How much will it hold at 40° if the mean coefficients of cubical expansion for glass and water between 4° and 40° are 0·00003 and 0·00027 respectively?

6. Describe some method by which the expansion of water has been studied. If S be the expansion of water between 4° and 0° and Δ its expansion between 4° and t°, show what is the density of water at t° referred to water at 0°. (L. '84.)

7. The height of a barometer as read by a brass scale at a temperature 15° was 760 mm. Find the true height reduced to 0°. The coefficients of cubical expansion of brass and mercury respectively are 0·0000552 and 0·000181.

CHAPTER VI

PANSION OF GASES. GAS THERMOMETERS

Pressure Coefficients.—Since the volume of solid or
depends but slightly on the pressure to which they are
ny pressure variations can be neglected when we are
their thermal expansion. This is not so for gases, as
shows, hence the effect of a rise in temperature on the
as is usually observed under two different conditions:
sure is kept constant and the alteration in volume is
(2) the volume is kept constant and the variation in the
neasured. In the first case we measure the coefficient
at constant pressure, or, more briefly, the volume
in the second we find the coefficient of pressure increase
volume, or the pressure coefficient. Experiment shows
efficients are much larger than any with which we have
preceding chapters, hence the approximate methods of
. 40) are no longer applicable; the increase in volume
must be compared with the volume or pressure, as the
, at 0° C. **The coefficient of expansion at constant**
fined as the ratio of the increase in volume for 1° rise
e to the volume at 0°, the pressure remaining constant.
olume at a temperature $t°$ and V_0 that at 0°, the co-
$\frac{V - V_0}{V_0 t}$, or $V = V_0(1 + \alpha t)$. Similarly the pressure
he ratio of the increase of pressure for 1° rise of tempera-
essure at 0°, the volume being kept constant. Denoting
d the pressures at 0° and $t°$ by P_0 and P respectively,
$\frac{P - P_0}{P_0 t}$, or $P = P_0(1 + \beta t)$.

at Constant Pressure.—Before describing the more
eriments of Regnault we will give two simple laboratory

Table showing α and β for Different Gases.

Name of gas.	Coefficient at constant pressure (α)	Coefficient at constant volume (β).
Air	0 00367	0 00367
Nitrogen	0·00367	0 00367
Hydrogen	0·00366	0 00366
Oxygen	—	0·00367
Carbon dioxide . . .	0 00374	0 00372
Sulphur dioxide . . .	0·00391	0 00386
Helium	—	0 00366

A glance at the table shows that the coefficient of expansion under constant pressure is practically the same for all gases. This was first noted by Charles, who expressed his results in the law known by his name: At constant pressure the coefficients of expansion of all gases are equal. As the table shows, this common coefficient is 0 00366 or 1/273. The law is not quite true as the numbers show; those gases which depart most widely from Boyle's law are more expansible than the law requires, *e.g.* carbon dioxide and sulphur dioxide. More extensive experiments establish the fact that at lower pressures or higher temperatures these gases also approximate to a condition in which both Boyle's and Charles' laws are obeyed. We are thus led to the notion of an ideal gas which obeys each of these laws accurately; such a substance is called a perfect gas. Although there is no substance which actually fulfils these conditions, yet for many purposes the more permanent gases like air, hydrogen, nitrogen, oxygen and helium may be treated as such. Accordingly in the following pages, unless otherwise stated, we shall regard these gases as perfect. Further reference to the above table also brings out the fact that for perfect gases the volume and pressure coefficients are equal.

Gas Thermometers.—The expansion of a gas at constant pressure may be used to measure temperature. In this respect it possesses various advantages over mercury; the expansion being larger it is more easily observed and the possibly irregular expansion of the bulb has less effect. In addition it could be used for much higher or lower temperatures, and two thermometers in which dif

' $= P_0(1 + \frac{1}{273} . t)$; let the temperature be supposed to decre
$-273°$C., then the pressure would be $P = P_0(1 - 1) = 0$, *provi*
as were perfect throughout this range. This temperature is the
he lowest that could possibly be read on such a thermomete
alled the **absolute zero of the perfect gas thermometer.** Ten
ures within a few degrees of this have actually been reacl
ecent years, but such extreme cold is found to liquefy or
solidify all gases when, of course, they cease to behave in the n
supposed. No stage is realised in practice at which the press
a gas is zero, nevertheless the idea of such a zero of tempera
found to be very useful. According to the kinetic theory t
molecules at this temperature are absolutely devoid of hea
therefore the lowest conceivable. The temperature of a
reckoned from this point as zero is called its **absolute temper**
on this scale ice melts at 273° absolute, water boils at 373° al
and generally, if T and t are the corresponding absolute and
grade temperatures of a substance, $T = 273 + t$.

Standard Gas Thermometer.—To measure the pressure w
apparatus shown in Fig. 34 four readings of mercury surfa
necessary, viz. those in B and C, and the zero and upper an
the barometric column. They are reduced to two in the s
gas thermometer shown in Fig. 36, and the possible error
halved. In this apparatus the vessel B of the simpler form
the reservoir of the barometer H, and the barometer tube
round so that the upper end of the mercury column is v
above the index at C. When the gas pressure in the bulb
that of the atmosphere the mercury stands at the same
e three limbs C, B, E, and the vertical distance CH is th
the barometer. As the temperature of the bulb A incre
servoir E is raised to keep the volume of the gas const
vel of the mercury therefore rises to B', E' and H' respe
e other tubes, and B'H' is now the barometric heig
is pressure is then CH' cms. of mercury, and this can i
easured by a cathetometer. The bulb A, about one
apacity, is made of an alloy of platinum and iridium;
municates with C through a very fine metal tube. W
esired to obtain a correction curve for a mercury thermo
ulb A and the thermometer are immersed in a well stirred
heir readings at different temperatures are compared; a
then be const

The absolute temperatures are 293° and 373°. Using the gas equation and calling v_2 the volume required,

$$\frac{75v_2}{373} = \frac{76 \times 1}{293}$$

whence

$$v_2 = 1{\cdot}23 \text{ litres.}$$

EXAMPLES ON CHAPTER VI

1. A litre of air at 0° and 76 cms. pressure weighs 1·293 gms. Find the weight of 5 litres when the temperature is 20° and the pressure 75 cms.

The weights of a litre of air under the two sets of conditions are proportional to the densities. Writing the gas equation in the form

$$\frac{P_1}{\rho_1 T_1} = \frac{P_2}{\rho_2 T_2} \quad \left(\text{since } v \propto \frac{1}{\rho}\right)$$

if x is the weight of a litre under the second conditions,

$$\frac{x}{1{\cdot}293} = \frac{\rho_2}{\rho_1} = \frac{P_2}{P_1} \cdot \frac{T_1}{T_2} = \frac{75}{76} \cdot \frac{273}{293}$$

$\therefore$ the weight of 5 litres $= 5x = 5 \times 1{\cdot}293 \times \frac{75}{76} \times \frac{273}{293}$

2. Determine the height of the barometer when a mgm. of air at 17° C. occupies a volume of 20 cms.³ in a tube over mercury, the mercury standing 73 cms. higher inside the tube than outside. [1 gm. of air at N.T.P. occupies 773·4 cms.³.] (L. '85.)

3. At the sea-level the barometer stands at 750 mm. and the temperature is 7°, while on the top of a mountain the barometer stands at 400 mm. and the temperature is −13°. Compare the weights of a cubic metre of air in the two places. The barometer readings may be taken as corrected for temperature. (L. '89.)

4. A given volume of air is at 740 mm. pressure at 17° C. What is its temperature when its pressure is 1850 mm.? (L. '03.)

5. State in symbols and in words the two laws which, if a gas obeys, it is called a perfect gas. One lb. of air at a temperature 0° and at a pressure of 1033 gms. per cm.² has a volume of 0·3555 cub. metres. At what pressure will its volume be 403,700 c.c. if measured at 27° C.? (L. '97.)

6. [illegible] how the apparent weight of a body in air varies with its [illegible] [illegible]. A piece of iron measuring 1000 c.c. is weighed at 0° and [illegible] What will be its apparent change in weight? Coefficient of expansion of air = 0·00367, of iron (linear) = 0·000012, mass of 1000 c.c. of air [illegible] [illegible] gms. (L. '86.)

7. Define the coefficient of increase of pressure of a gas. Show that, if a gas obeys Boyle's and Charles' laws, this coefficient is equal to the coefficient of expansion. (L. 1900.)

8. Find the number of feet in a steel bottle to hold at 120 atmospheres pressure, when the temperature is 25° C., 20 cub. ft. of oxygen under normal conditions. (L. '03.)

9. The mercury in a barometer containing some air stood at a height of 70 cms and the volume of the tube above the mercury was 20 c.c. The tube was then lowered into the reservoir until the volume above the mercury was 10 c.c., when the barometer indicated 65 cms. only. Calculate (1) the true barometric height, and (2) what the reading of the barometer in question would be if its tube were raised until the volume above the mercury became 100 c.c. (L. '06.)

10. A sample of gas was found to have a volume of 100 c.c. at 15° and 72 cms. pressure, and a volume of 200 c.c. at 90° and 45 cms. pressure. Assuming that the gas obeys Boyle's law and expands uniformly at constant pressure, calculate at what temperature it would have a volume of 400 c.c. at 100 cms. pressure. (L. '07.)

11. A volume of 50 c.cs. of air at 15° is expelled from the bulb of a constant pressure air thermometer by changing the temperature from 0° to 100° C. Given the coefficient of expansion of air is 1/273, calculate the temperature of the thermometer when 10 c.cs. are expelled, neglecting the expansion of the bulb. (L. '05.)

CHAPTER VII

CHANGE OF STATE

Melting and Boiling Points.—When a solid like ice or paraffin wax is continually heated a temperature is finally reached at which it liquefies; this temperature is called the melting point of the substance. Provided the pressure is unchanged a substance always melts at the same temperature, this provides the chemist with a means of identification. Similarly when a liquid is continually heated a stage is reached where the temperature remains steady and the liquid is continuously converted into vapour.[1] The liquid is then said to boil. The temperature at which boiling takes place is called the boiling point. It also is characteristic of the substance but varies greatly with the pressure. These changes can take place in the inverse order; thus if steam is cooled it finally condenses to liquid, and the water so formed, if its temperature is sufficiently reduced, at last solidifies or freezes. Except when chemical change is produced the condensing point coincides with the boiling point and the freezing point with the melting point. Certain substances, such as glass, have no well-defined melting point, in changing from the solid to the liquid state they pass through an intermediate pasty condition; it is this property which makes it possible to work with glass in the blow-pipe flame. Other substances, such as iodine, when heated pass directly from solid to vapour without becoming liquid; they are said to sublime. The reverse change, directly from vapour to solid, occurs in the case of hoar frost

EXPERIMENT.—Put a quantity of melting ice in a beaker, in a second beaker an equal weight of cold water, and place each of them over Bunsen burners [illegible] and that the temperature of the water rises at once [illegible], but the temperature in the other beaker remains steady [illegible] is melted, after which it increases as in the other vessel.

[1] A vapour may be defined as the gaseous form of a liquid.

This is a typical case of melting; the temperature of the melting substance is constant during the process. As heat is entering from the flame it is clear that the solid absorbs heat without changing its temperature; this heat is said to be latent.

The number of calories required to convert one gram of a solid into a liquid without changing its temperature is called the latent heat of fusion of the substance.—For example, the latent heat of fusion of ice is 80 calories, i.e. the heat necessary to liquefy one gram of ice would raise the temperature of a gram of water from 0° to 80°. Other solids behave in a similar manner, but the actual value of the latent heat of fusion varies with the substance. Before a liquid at the freezing point can solidify it must part with its latent heat of fusion, while it is doing this its temperature remains steady; upon this fact is founded a method of determining the freezing point (p. 71). Similar absorptions or evolutions of heat are shown at the boiling point.

EXPERIMENT.—Heat over Bunsen burners two small flasks, one containing mercury, the other water. When 100° C. is reached the water is gradually converted into steam and its temperature is constant, that of the mercury steadily rises beyond this point. Heat is absorbed in each case, that entering the mercury causes a rise in temperature, while that absorbed by the water at 100° becomes latent; this latent heat is used in producing steam. Finally at a temperature near 350° the mercury also boils and absorbs latent heat.

The number of calories required to convert one gram of a liquid into vapour without changing its temperature is called the latent heat of vaporisation of the liquid.

Methods of determining the Melting Point.—When a substance shows a clearly defined melting point, or is obtainable only in small quantities, the following method may be used.

EXPERIMENT.—A few small particles of the substance are placed in a thin-walled capillary glass tube which is attached to a thermometer bulb by rubber bands. This is mounted in a test tube and placed in a beaker of water as in Fig. 37. The water is slowly heated and the temperature at which the substance melts is observed; the whole is then allowed to cool and the temperature of solidification noted. These observations are repeated until the two temperatures differ by only a few tenths of a degree, when their mean is taken as the melting point. The air currents in the test-tube ensure that the bulb and capillary tube are at the same temperature.

Another method is used where it is difficult to tell by the eye when melting actually takes place. The substance is thoroughly melted

and allowed to cool slowly, the temperature being observed every 10 secs., a cooling curve is then plotted showing the temperature at different times. When solidification begins the liquid gives out its latent heat of fusion and the temperature remains steady for some time, this is shown clearly on the cooling curve. The method is largely used by metallurgists to find the freezing points of metals; as the temperatures are much higher than can be read by a mercury thermometer a thermo-couple is used in place of it. (See p. 431.)

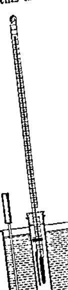
FIG. 37.—Apparatus to determine Melting Points.

EXPERIMENT.—Some tinman's solder was melted in a large crucible and a hard-glass test-tube containing mercury was pushed into it. A thermometer was placed in the mercury and the cooling curve shown in Fig. 38 was obtained. The mercury provided good contact with the solder and yet preserved the thermometer from breakage when solidification took place. Owing to solder containing two metals, freezing takes place in two steps at temperatures near 194° and 178°. The student should obtain by this method the melting point of paraffin wax.

The curve shows that it is possible to cool a liquid below its freezing point without causing it to solidify, but directly solidification begins the temperature rises to that of the normal freezing point and remains steady until the change of state is completed (at 194° in Fig. 38). A substance cooled below its freezing point, the remaining fluid, is said to be supercooled. The "hypo" used by photographers shows supercooling exceedingly well.

EXPERIMENT.—Powder some "hypo" in a mortar and place it round the bulb of a thermometer in a test tube. Heat it in a beaker of water until it melts in its own water of crystallisation; this takes place at about 48°. The test tube may then be removed from the beaker and allowed to cool, a cooling curve being obtained in the usual way. If it is not shaken the temperature [illegible] ly when [illegible] is steady [illegible] the "hypo" solid [illegible] [illegible] y to [illegible] into [illegible] makes it [illegible]

...owed to cool slowly, the temperature being observed every ..., a cooling curve is then plotted showing the temperature at ...t times. When solidification begins the liquid gives out its ...heat of fusion and the temperature remains steady for some ... this is shown clearly on the cooling curve. The method is largely used by metallurgists to find the freezing points of metals; as the temperatures are much higher than can be read by a mercury thermometer a thermo-couple is used in place of it. (See p. 431.)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Experiment.—Powder some "hypo" in a mortar and place it round the bulb of a thermometer in a test-tube. Heat it in a beaker of water until it melts in its own water of crystallisation; this takes place at about 48°. The test-tube may then be removed from the beaker and allowed to cool, a cooling curve being obtained in the usual way. If it is not shaken the temperature may fall to 30° without the "hypo" solidifying. Finally when solidification begins the temperature rises suddenly to 48° and remains steady for some minutes. If a solid crystal is dropped into the supercooled liquid solidification at once. The absence of dissolved air from a liquid makes it more ...ble of being supercooled.

Latent Heat of Fusion.—The latent heat of fusion of ice can be

Also heat lost by the water originally in the calor-
ter $= m_1(t_1 - t_2)$ cals.

And heat lost by the calorimeter $= m_2 s_2(t_1 - t_2)$ cals.

Hence $ML + Mt_2 = m_1(t_1 - t_2) + m_2 s_2(t_1 - t_2)$, from which
n be found. Accurate experiments show that L for water is ve
arly 80 cals. If the ice is not thoroughly dry the water added wi
will not absorb its latent heat and the final result will be too lo
o render the radiation correction small the calorimeter should
above the room temperature and sufficient ice should be add
o cool it by an equal amount below.

Change of Volume produced by Melting.—Since ice floats in water
at 0° its specific gravity must be the smaller of the two; in other words, a c.cm. of water at 0° will expand to more than a c.cm. when it freezes. It is due to this expansion that water-pipes are frequently burst during a frost. On the other hand solid paraffin wax sinks when thrown into its liquid at a temperature just above the melting point. Paraffin therefore contracts when it solidifies. Substances which are to be cast should expand at the moment of solidification in order to retain the shape of the mould.

EXPERIMENT.—*To find the Specific Gravity of Ice.* Pour about 50 c.cms. of methylated spirit into a small beaker and drop into it a small lump of ice. Add water until the ice is nearly wholly immersed. Stir the mixture; ice gradually melts, at the moment it sinks below the surface remove it as quickly as possible. By mixing two liquids, one having a greater the other a less specific gravity, a mixture has been made in which the ice floats, its specific gravity, which equals that of ice, may now be found by means of a specific gravity bottle.[1]

According to Bunsen 1 gm. of ice at 0° occupies 1·0908 cm.^{3}, and a gm. of water at the same temperature has a volume 1·0001 cm.^{3}. The expansion when a gm. of water freezes is therefore 0·0907 cm.^{3}.

Bunsen's Ice Calorimeter.—Bunsen has utilized this volume change in the construction of a very delicate calorimeter. A tube P (Fig. 39) is fused into the upper end of a wider tube Q, shaped as shown in the figure, and the space between them is filled with water from which the dissolved air has been removed by boiling. Mercury is then poured in until it rises to the top of the narrow tube at S. R is a capillary tube, graduated in c.cms., which is pushed through a rubber stopper at S until the mercury extends to near its middle. A block of ice is next made to form round the bottom of the

[1] Barton and Black, "Practical Physics," p. 34.

tube P. With this object the apparatus is placed
the water is free from air it may be greatly supercooled
begins. To start the freezing a little ether is plac
caused to evaporate quickly by bubbling air thr
evaporating liquid absorbs its latent heat of vaporisation from the water and so causes it to freeze; once begun this will continue for some hours. When sufficient ice has been formed, water cooled down to 0° is placed in P, and the hot body whose specific heat is required is dropped into it after noting the position of the mercury thread. It cools from T° to 0° and emits MsT calories of heat, thereby melting some ice. The volume of the water in Q is thus altered and the change is read off on the

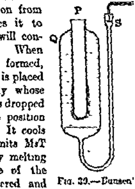

FIG. 39.—Bunsen'

graduated tube R. Let v be the volume change, t
of gms. of ice melted is, from the last paragraph
the heat it absorbs is 80v/0·0907, since the latent h

Hence
$$MsT = \frac{80v}{0.0907}$$
nd
$$s = \frac{80v}{0.0907MT}$$

Instead of bringing into the calculation the late
uantity about which there is some uncertainty, the
e standardised by pouring into the tube P a mass
t a temperature t°. The heat it emits in cooling
f this causes the mercury in R to move over n d
livision corresponds to an emission of mt/n cals.
mitted in any subsequent experiment is known fro
of the mercury column.

Solution. Freezing Mixtures.—When a solid
solved in a liquid it absorbs its latent heat of fusion
and, unless chemical actions occur, the temperature

again, giving out its latent heat. This heat, if transmitted through the wire, will assist the pressure in melting more ice; the wire thus works its way through the block which nevertheless remains whole. If an iron wire is substituted for copper its rate of progress is slower owing to its being a worse conductor of heat (p. 119). It is due to regelation also that skating on ice is possible; the pressure of the steel edge causes ice to melt and so allows the skate to "bite." Similarly the lower portions of glaciers melt under the great pressure to which they are subjected. In the case of ice the lowering of the melting point is very small, about 0.0072° per atmosphere, hence it is unnecessary to allow for variations in the barometric height when the lower fixed point of a thermometer is being found.

Boiling Point. Latent Heat of Vaporisation.—When liquid is heated in a beaker bubbles of air and vapour of the liquid are formed on the glass which finally rise to the surface and burst, causing a sound. The "singing" of a kettle is due to this. When a certain temperature is reached the supply of bubbles is very copious and the temperature remains steady; the liquid is then said to boil. If it has been previously freed from dissolved air its temperature may rise above the normal boiling point before it actually commences to boil, it is then said to be superheated; finally a bubble of air or vapour is formed and violently bursts. This "bumping" may be hindered if a supply of air bubbles is provided by putting into the liquid some broken pieces of earthenware. As the temperature of a boiling liquid depends slightly on the vessel in which it is contained, the boiling point is determined by a thermometer whose bulb is placed in the vapour above the liquid. The latent heat of vaporisation is most easily determined by Berthelot's apparatus (Fig. 41). The liquid is heated in a special form of flask through the bottom of which projects a glass tube open at both ends, this is connected by a ground joint to a glass bulb and spiral immersed in water in a calorimeter. The calorimeter and its contents are protected from heat coming from the gas burner by a wooden cover, and a stirrer and thermometer are passed through holes in the wood. When the liquid boils its vapour passes down the vertical tube and is condensed in the spiral, at the same time giving up its latent heat. The amount condensed may be found by weighing the spiral at the beginning and the end of the experiment. As the vapour passes down through the upper part of the tube any drops of liquid that it carries with it are vaporised, if this did not happen the particles

t this temperature change is entirely due to the act of solution solid must be initially at the same temperature as the solvent.

EXPERIMENT.—Place some powdered "hypo" in a test-tube and immerse for 30 mins. in water contained in a calorimeter. Note the temperatures of solid d liquid should then be equal. Note the temperature of the water and pour e contents of the test-tube into the calorimeter; the temperature falls. It is r this reason that the fixing solution used in photography should be made up me time before it is required for use, otherwise its temperature will be low and s action correspondingly slow.

When snow and salt are mixed together they mutually dissolve each other, and, in accordance with what has just been said, a very low temperature results. This is the principle of freezing mixtures. After a fall of snow salt is frequently thrown on the pavements to make the snow melt; it produces the attendant discomforts of a slush whose temperature is below 0°. If 33 parts by weight of sodium chloride are mixed with 100 parts of ice a temperature as low as − 20° C. can be reached.

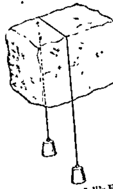

FIG. 40.—Tyndall's Experiment on Regelation.

Effect of Pressure on the Melting-Point.—Since ice contracts in volume when it melts we should perhaps expect that an increased pressure, which renders a contraction more liable to take place, would cause it to melt more easily, or, in other words, would cause it to melt at a temperature below 0°. Similarly when a substance expands on melting it is possible that an increased pressure would raise the melting point. This, in fact, is what actually takes place. When a strong steel cylinder at 0° is filled with pieces of ice and these are subjected to great pressure they partially melt, if the pressure is then relieved the water freezes, since its temperature is 0°, and the whole is found to have formed a solid block of ice. This effect of pressure is called regelation. The following experiment, due to Tyndall, may be explained in the same manner. Weights are hung over a block of ice by means of a copper in where the temperature is 0° (Fig. 40). Owin the wire ice melts, and the water which is side of the copper, where, being re

would not have to part with their latent heat to the calorimeter, and the final result obtained would be too low. Let M be the weight of liquid condensed, T° its boiling point, L its latent heat of vaporisation, and s its specific heat. Let m be the total water equivalent of the calorimeter and its contents, including the spiral and stirrer, t_1° its initial and t_2° its final temperature.

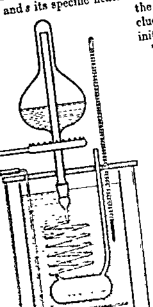

FIG. 41.—Berthelot's Apparatus.

Then the heat given out by the vapour in condensing = ML cals. and the heat emitted by M gms. in cooling from T° to t_2°

$$= Ms(T - t_2) \text{ cals.}$$

Also the heat absorbed by the calorimeter and its contents

$$= m(t_2 - t_1) \text{ cals.}$$

Hence

$$ML + Ms(T - t_2) = m(t_2 - t_1)$$

from which L can be found if the specific heat of the liquid and its boiling point are known. The latent heat of vaporisation of water is 537 calories. This large latent heat is turned to a useful purpose in the heating of buildings by steam. Evidently much less material is required than if hot water alone were used.

EXPERIMENT.—Drop a little ether on the hand and note that a cooling sensation is experienced. The liquid is very volatile and evaporates very quickly as due that it absorbs its latent heat of vaporisation.

Joly's Steam Calorimeter.—Prof. Joly in his steam calorimeter has worked out a very simple and accurate method of determining specific heats which depends on a knowledge of the latent heat of steam. A simple form of the calorimeter is shown in Fig. 42. It consists of a metal enclosure, called the steam chamber, into which a rapid supply of steam can be admitted through a wide tube and ... at the bottom of the chamber the exit tube is placed. ... the top ... a vertical wire passes through a small hole at the top and ...

attached at its upper end to one arm of a balance. The body wh[ose] specific heat is required is placed on a small copper pan which ha[ngs] from the lower end of the wire in the middle of the chamber[;] thin copper guard shields it from drops of condensed water wh[ich] might otherwise fall on to it from the roof of the enclosure. T[he] substance is allowed to hang in the calorimeter for some minut[es] and its temperature t_1 is then taken.

Steam is now admitted through the wide tube and condenses on the body and pan; after a few minutes the mass condensed is found from the increased weight of the pan and its contents. Let m be the mass of steam which condenses, m_1 and m_2 the masses of the substance and the pan, s_1 and s_2 their specific heats, and L the latent heat of steam. The enclosure is finally at the temperature t_2 of the steam, hence the heat given out during condensation is mL, and the heats absorbed by the body and pan respectively are $m_1s_1(t_2 - t_1)$ and $m_2s_2(t_2 - t_1)$. Therefore

$$mL = m_1s_1(t_2 - t_1) + m_2s_2(t_2 - t_1)$$

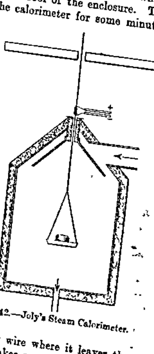

FIG. 42.—Joly's Steam Calorimeter.

The specific heat s_1 can be calculated from this equation if s_2 is determined by a preliminary ex[-]periment. In practice it is found that steam condenses on the suspending wire where it leaves the [s]team chamber; surface tension then makes an accurate weighing [i]mpossible. To overcome this difficulty the wire is passed along the [ax]is of a small spiral of platinum which is heated by passing an [e]lectric current through it; sufficient heat is thus developed to [h]inder condensation. When a liquid is to be experimented on it is [e]nclosed in a small copper sphere. The most novel application of [th]e apparatus was in the determination, for the first time, of the [sp]ecific heats of gases at constant volume. For this purpose two [eq]ual copper spheres were hung from the opposite arms of the

If the latent heat of fusion of ice is 80 and its density at 0° is 0·917,
the travel of the mercury in the tube of a Bunsen's ice calorimeter when 10
are given to the ice, the diameter of the tube being 0·4 mm. (L. '08.)

9. The boiling point of a liquid is 150°, its mean specific heat is 0·46 and
latent heat is 68 gm. cals. Find the quantity of vapour at the boiling p
that must be passed into a copper vessel (sp. ht. 0·1) weighing 30 gms., wh
contains 250 gms. of the liquid at 15°, in order to raise the temperature of
latter to 27°. (L. '09.)

CHAPTER VIII

VAPOUR PRESSURE. CHANGE OF STATE (*continued*)

Vapour Pressure.—A solid changes into liquid at one temperature only, the melting point, but a liquid can assume the form of vapour at any temperature. Thus a pool of water on the road dries up under the sun's rays; although the temperature is below the boiling point the water evaporates, *i.e.* is converted into vapour.

EXPERIMENT.—Set up a barometer and introduce a small drop of alcohol at the lower end of the tube by means of a curved pipette; the bubble of liquid [illegible] up the column and is at once converted into vapour when it reaches the surface. The vapour exerts a pressure just as a small quantity of air would do and the mercury is depressed by an amount equal to the vapour pressure. If additional alcohol is introduced more vapour is formed and the height of the column is depressed still further, but at length a stage is reached when the added liquid [illegible] on the surface and no further evaporation takes place.

[illegible] the maximum amount of vapour it can hold [illegible] conditions of temperature the vapour is said to be [illegible] pressure it exerts is called the saturated or maximum [illegible]. If less than this maximum amount [illegible] be unsaturated or superheated. The [illegible] sometimes used instead of [illegible] vapour [illegible]

[illegible] different diameters and lengths [illegible] is varied; it will be found that [illegible] the maximum depression is [illegible] that [illegible] provided the temperature is constant.

[illegible] are used it will be found that the maximum [illegible] one to another; also when the temperature [illegible] evaporates and the maximum vapour pressure [illegible] results show that to every liquid there corresponds [illegible] pressure varying with the nature of [illegible]

substance but otherwise depending on the temperature alone. Whe[n] an unsaturated vapour is gradually cooled a point is reached whe[n] its pressure is equal to the maximum vapour pressure correspondin[g] to that temperature; the vapour is then saturated and any furthe[r] cooling is accompanied by partial condensation. Similarly if an unsaturated vapour is compressed at constant temperature, as, for example, by pushing down the barometer tube containing it into a deep cistern of mercury, a state of things is eventually arrived at where the vapour actually present is sufficient to saturate the space, further compression then causes condensation but *the vapour pressure remains constant*.

FIG. 43.—Hofmann's Vapour Density Apparatus.

Vapour Density.—Unsaturated vapours obey Boyle's and Charles' laws very approximately if the temperature is well above that at which condensation would begin. This is best proved by measurements of the vapour density at different pressures and temperatures. The method is to find by experiment the mass of vapour in a c.cm.—this is the density as usually defined; the volume v of a gram of vapour can then be calculated under the pressure and temperature prevailing in the experiment. It will be found, as in the case of gases, that $\frac{pv}{T}$ = constant. One method of experiment is shown in Fig. 43. A long barometer tube graduated in c.c.'s contains mercury and is surrounded by a steam jacket. A known weight of liquid enclosed in a small stoppered bottle is placed in the open end of the tube and rises to the top of the mercury column where, owing to the diminished pressure and the high temperature, the stopper is ejected and the liquid forms an unsaturated vapour. The depression of the mercury column measures the vapour pressure; this may be found at once by a cathetometer. The volume is read off from the graduations and the temperature is given by a thermometer in the steam jacket, hence as the mass of the vapour is known, being equal to the liquid introduced, its density can be calculated. Using different amounts of the substance, and the vapours of different

... in the upper steam chamber. One sphere contained t...
...ant mass of gas while the other was exhausted; the difference...
... weights of steam condensed was thus due to the contained ...
... specific heat of air at constant volume under a mean pressure ...
... 19.5 atmospheres was found to be 0.1721, hence from the t...
... p. 31 $C_p/C_v = 1.4$

... $= 1529$

EXAMPLES ON CHAPTER VII

1. The open end of the capillary of a Bunsen calorimeter is placed under the surface of mercury. When 25 gms. of water at 15° are placed in the inner tube of the calorimeter it is found that 6.9 gms. of mercury are drawn in. Assuming the density of mercury to be 13.6 and the latent heat of ice as 79, determine the density of ice. (L. '90.)

2. A calorimeter whose capacity for heat is 44 water gms.-degrees has 352 c.c. of water in it and the whole weighs 442 gms. Into this steam at atmospheric pressure is condensed till its temperature rises from 12.2° to 18.7°, and on weighing again the calorimeter weighs 446.2 gms. Calculate the latent heat of vaporisation of water.

3. If a boiler receives 30,000 units of heat per min. through every square metre of its surface, the total surface being, say, 8 sq. m., and if its temperature be 140° while it is fed with condenser water at 45°, what weight of steam would you expect to be able to draw off regularly per hour? The latent heat of vaporisation of water at 140° is 500. (L. '91.)

4. One hundred gms. of iron at 60° C. are placed in a vessel containing 1000 gms. of water at 0°; how many gms. of ice at 0° must be added to reduce the temperature of the mixture to 0°? All the ice is supposed to be melted. [Sp. ht. of iron = 0.113; lat. ht. of fusion of ice = 80.] (L. '93.)

5. One gm. of metal heated to 100° is dropped into a Bunsen ice calorimeter in which the weight of mercury required to fill 1 cm. of the index tube has been found to be 0.026 gm. The thread of mercury moves through 52.5 mms. What is the mean specific heat of the metal? One gm. of water in freezing expands 0.0907 c.c. and its latent heat of fusion is 80.02. The density of mercury is 13... (L. '02.)

6. A mass of 200 gms. of copper (sp. ht. 0.1) is hung in a closed chamber at a temperature of 60° F. Steam is then admitted at the normal atmospheric pressure. Calculate the mass of water condensed by the copper. [Lat. h... steam = 536.] (L. '03.)

[illegible] into [illegible] weighing 1.0... [illegible]

7. Steam at [illegible]
and conta[illegible]
and its [illegible]

substance but otherwise depending on the temperature alone. When an unsaturated vapour is gradually cooled a point is reached when its pressure is equal to the maximum vapour pressure corresponding to that temperature; the vapour is then saturated and any further cooling is accompanied by partial condensation. Similarly if an unsaturated vapour is compressed at constant temperature, as, for example, by pushing down the barometer tube containing it into a deep cistern of mercury, a state of things is eventually arrived at where the vapour actually present is sufficient to saturate the space, further compression then causes condensation but *the vapour pressure remains constant*.

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CHAPTER VIII

VAPOUR PRESSURE. CHANGE OF STATE (continued)

Vapour Pressure.—A solid changes into liquid at one temperature only, the melting point, but a liquid can assume the form of vapour at any temperature. Thus a pool of water on the road dries up under the sun's rays; although the temperature is below the boiling point the water evaporates, i.e. is converted into vapour.

EXPERIMENT.—Set up a barometer and introduce a small drop of alcohol at the lower end of the tube by means of a curved pipette; the bubble of liquid rises up the column and is at once converted into vapour when it reaches the surface. The vapour exerts a pressure just as a small quantity of air would do and the mercury is depressed by an amount equal to the vapour pressure. If additional alcohol is introduced more vapour is formed and the height of the column is decreased still further, but at length a stage is reached when the added liquid floats on the surface and no further evaporation takes place.

When a space contains the maximum amount of vapour it can hold under the given conditions of temperature the vapour is said to be saturated and the pressure it exerts is called the saturated or maximum vapour pressure. If less than this maximum amount is present the vapour is said to be unsaturated or superheated. The term "vapour tension" is sometimes used instead of "vapour pressure."

EXPERIMENT.—Use barometer tubes of different diameters and lengths so that the volume occupied by the vapour is varied; it will be found that while more liquid is evaporated in the larger tubes the maximum depression is the same in every case provided the temperature is constant.

If different liquids are used it will be found that the maximum pressure varies from one to another; also when the temperature is raised more liquid evaporates and the maximum vapour pressure ... These results show that to every liquid there corresponds a ...um vapour pressure varying with the nature of t...

at BC, is parallel to the axis of volume. At C all the substance has condensed, and, as the volume of a liquid varies very little with pressure, the remaining part CD of the curve is very nearly parallel to the pressure axis. Summarising these results we see that along AB the substance is wholly vapour, along BC liquid and vapour in contact, and along CD wholly liquid.

Methods of measuring Maximum Vapour Pressure.—The methods used to measure the maximum vapour pressure vary with the temperature, a procedure which is useful at 20° may be inconvenient at 80°. The apparatus shown in Fig. 45 was used by Regnault to measure the vapour pressure of water below 0°. Bulb A, which contains the water, forms the upper part of a barometer tube; it is placed in a freezing mixture of calcium chloride and snow. The vapour pressure is the same at all points in this space and is equal to the maximum pressure corresponding to the temperature of the freezing mixture; if it were higher than this condensation would take place in A. On the left is shown an ordinary barometer; the difference in heights of the two columns gives the vapour pressure in cms. of mercury. The vapour pressure of ice, which is quite appreciable, can be measured by this means.

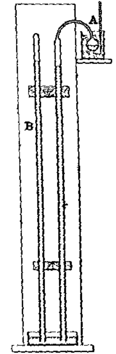


FIG. 45.—Regnault's Apparatus for measuring the Vapour Pressure of Water below 0°.

Fig. 46 shows Regnault's apparatus for water between 0° and 50°. One vertical tube forms a standard barometer, in the other a little water floats above the mercury. The upper part of each tube is surrounded by a water-bath which is kept well-stirred and can be heated from below. The difference in heights of the two columns is read by a scale, or, in Regnault's experiments, by a cathetometer; this gives the vapour pressure at the temperature of the bath in cms. of mercury. As the mercury is warm the pressure must be given in terms of the length of a mercury column at 0°

g liquids in the steam-jacket, it can be proved that pv/T onst., where v is the volume of one gm. of vapour at a ure p and temperature (absolute) T.
imilar experiments in which different substances are used he same temperature, bring out further the important fact the vapour density is proportional to the molecular weight o substance used. This result is important from the chemica

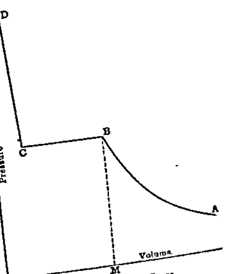


FIG. 44.—Isothermal Curve.

point of view as it enables molecular weights to be found fro vapour density determinations.

Isothermal Curves.—We can now show the general shape of isothermal curve which gives the relation between the pressure a volume of a substance at a constant temperature. Starting w the vapour in the unsaturated state $pv =$ const. until saturatio nearly reached; this part of the curve is shown at AB (Fig. When the volume is reduced to the amount represented by condensation begins and the vapour pressure remains constant all the vapour is converted to liquid. This part of the curve, al

el and a thermometer (Fig. 48). The funnel contains the liquid e experimented on and the lower end of its stem is drawn off to ne point which is near the thermometer bulb. The latter i pped round with cotton wool or asbestos. In communicatio h A is a bottle B immersed in ice, a large bottle C and a mano ter M; the tube F goes to an air or filter pump. A convenier essure having been established by the pump tap D is closed. Th th surrounding the boiling tube is then heated to a temperature w degrees higher than the boiling point of the liquid under th ven pressure, and liquid is allowed to drip slowly on to the therm eter bulb. As a large surface is exposed boiling takes pla uite regularly; when the thermometer reading is steady t emperature and pressure are read. This gives the vapour press t the temperature shown by the thermometer. By varying pressure a series of measurements of the vapour tension at differ temperatures can be found. The bottle B is for the purpose condensing the vapour so that the liquid may be recovered, while serves to lessen the pressure variations due to accidental causes suc as a slight leakage of air into the apparatus. If the vapour pressu is greater than that of the atmosphere air must be compressed in the apparatus which should then be made correspondingly stronge In this form it may be used for water above 100°. This dynamic method is much more accurate and easier to work than the s given in the last paragraph which is usually called the stati method.

Vapour Pressure of Salt Solutions.—The dynamical method already described cannot be used to determine the vapour press of a salt solution because as the liquid boils off the concentration the solution is altered. The statical method (p. 87) is, howe available, and another arrangement (p. 103) is also frequently u If some salt solution is introduced into the space above a barom column it is found to produce a smaller depression of the mer than the pure liquid does at the same temperature, hence vapour pressure of a solution is less than that of the pure sol It follows that at 100° the vapour pressure of an aqueous sol will not be equal to the atmospheric pressure and the liquid m heated to a higher temperature to make it boil. This has al been noted on p. 22. As the vapour leaves the liquid it cools quickly to 100°, hence to find the boiling point of a solutio thermometer bulb must be placed in the liquid itself and not in t

vapour. The vapour pressure of volatile liquids like ether and alcohol is much greater than that of water and becomes equal to the atmospheric pressure at temperatures below 100°. Such liquids have consequently a low boiling point, *e.g.* ether boils at 34.5° under normal pressure.

Determination of Heights by the Hypsometer.—The pressure at a point in a barometer tube becomes less as the point in question is taken nearer the top of the mercury column. In the same way, during the ascent of a mountain, as the different layers of air are passed through and the limits of the atmosphere are more nearly approached the pressure becomes less and the length of the barometric column is reduced. The height of the mountain can be calculated if the barometric pressure at its summit is measured. Instead of using a barometer for the purpose the temperature at which water boils may be observed, and from a table of maximum vapour pressures the corresponding pressure of the atmosphere can be found. An instrument used for this purpose is called a hypsometer. At great altitudes the boiling point may be lowered to such an extent that it is impossible to cook food. An arrangement for boiling under increased pressure must then be employed.

Dalton's Law for Mixed Vapours.—Let us next investigate how the pressure of a saturated or unsaturated vapour is modified by the presence of a gas or other vapour with which it does not react chemically. According to Dalton the total pressure produced by such a mixture is the sum of the pressures that each component would produce if it alone were present. This is usually known as Dalton's law. It is only approximately true in most cases; if it held in every instance it would be possible to produce a pressure as great as we pleased by introducing a sufficient number of different components into the mixture. Regnault tested the law by means of apparatus similar to that in Fig. 46.

EXPERIMENT.—Having set up a barometer in the usual manner introduce a small quantity of air. Suppose the column is depressed h cms. and let the total length of tube occupied by the air be L cms. Next add ether until the space above the mercury is saturated, and suppose the total depression is H cms. Then $(H - h)$ does not measure the vapour pressure of the ether, for the air is now diffused through a larger volume and its pressure is therefore less than h. Call the new air pressure h_1. Let L' cms. of the tube be occupied by air and ether vapour and suppose the sectional area of the tube is S. The volumes of

ir before and after admission of the ether are LS and L'S c.cms. respectively, the corresponding pressures are h and h', hence by Boyle's law

$$h'L'S = hLS$$
$$h' = hL/L'$$

ng the new pressure of the air. The pressure of the mixture being H cms. t of the ether is $H - h' = H - \frac{hL}{L'}$. Working in this manner it will be found at the maximum vapour pressure of ether is the same as it would have been in e absence of air, thus proving Dalton's law.

An easier method of performing the experiment is indicated ir ig. 49. The flask contains air and a small, closed, thin-walled bulb lled with ether. Having noted the air pressure on the manometer he flask is shaken to break the bulb, and after some minutes the new pressure is noted. The section of the manometer tube being small we may suppose the volume of the air constant, the increase in pressure is thus due to the ether alone; this will be found equa to the maximum vapour pressure of the liquid at the temperature o the experiment, if some liquid ether still remains in the flask It will be noticed that the ether evaporates much more slowly whe another gas or vapour is present.

Cooling produced by Evaporation.—During evaporation it is onl the more rapidly moving molecules that escape from the liquid surfac to form vapour. The average velocity of the remaining molecul is thus reduced, or, in other words, the liquid is cooled. This merely another method of stating that the latent heat of vaporis tion is absorbed when a liquid evaporates. If the space above crowded with air molecules it will be more difficult, owing to collisio for molecules to escape from the liquid, this accounts for t relative slowness with which evaporation takes place in presence a gas. The cooling produced by evaporation may be shown various ways.

Experiment.—Dip a thermometer in ether and note the temperatur It is then removed the adherent film of liquid evaporates and the thermo temperature falls.

Experiment.—The bulb F (Fig. 50) contains ether and is connected rom which the air has been removed before sealing. Wh her vapour is condensed and more evaporates from the m a cooling is thus produced which is readily shown A (Fig. 17) of the Lower thermoscope. If the bulb ether the fall in temperature may be sufficient to caus ter form the apparatus is called Wollaston's cryophor

EXPERIMENT.—Pour a little water into a shallow depression in a w
block and place on it a small copper vessel containing ether. If the et
evaporated quickly by blowing a current of air through it the cooling pro
may be great enough to freeze the water

In the case of ponds, lakes, etc, evaporation is continuo
taking place from the surface, at least in the summer months F
what has been said in the preceding pages it will readily be
that the conditions favourable to the process are: (1) A high temp
ture, (2) Little vapour already present in the air; (3) The vapour n
be removed by air currents as rapidly as it is formed; (4) A l
surface

Condensation. Liquefaction of Gases.—Since an unsatura

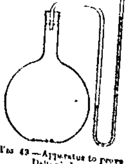


FIG 49 —Apparatus to prove Dalton's Law

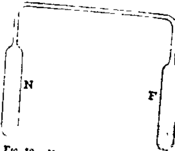


FIG 50 —Method of showing the Cool
produced by Evaporation.

vapour becomes saturated if the temperature or volume is sufficiently
reduced, any vapour may be made to condense into liquid by one o
other of these processes or a combination of the two. Steam escaping
rom a jet is invisible near the orifice where it is purely vapour, but
at a greater distance it becomes cooled and condenses into a large
number of small particles of water which form a readily visible cloud.
Not only vapours but gases also may be liquefied by great pressure
if the temperature is low enough. Faraday liquefied a number of
gases by such means in 1823 Chlorine may be taken as a typical
example Charcoal absorbs a large amount of chlorine gas; some
charcoal saturated with chlorine is placed in one limb of a bent glass
the which is then closed at both ends and the other limb is surr
ounded by a freezing mixture. Gas is evolved by heating the

equal at the temperature of the room. Will they still be equal when the perature is raised or lowered, and if not which will give the higher readings? 93.)

2. A bubble of air is stuck on the side of a vessel in the interior of a mass of id. Show that its volume tends to become very great as the boiling point the liquid is approached.

3. Describe carefully the difference between evaporation and boiling. What ect has the presence of air above the liquid in each case? Why does ether l at a lower temperature than water? (L. '97.)

4. A barometer tube dipping into a mercury reservoir contains a mixture f air and saturated vapour above a column of mercury which is 70 cms. above he tube in the reservoir, the atmospheric pressure being 76 cms. What is the eight of the mercury column when the tube is depressed so as to reduce the volume occupied by the air to half its original value, the pressure of the saturated vapour being 1·5 cms.? (L. '08.)

5. A quantity of air saturated with aqueous vapour occupies a volume 120 c.cs. at 18° under a pressure of 77 cms.; the pressure is increased to 150 cm the temperature remaining constant, and the volume is found to be halve Find the vapour pressure.

CHAPTER IX

HYGROMETRY

Relative Humidity.—In popular language we frequently speak of the atmosphere as dry or moist, but it is easy to see that our sensations may lead us into error concerning its physical state. Thus on a summer morning when there is a slight mist and dew we say the air is moist, while later in the day we call it dry, in spite of the fact that it then contains more water vapour owing to the evaporation of the particles of dew. We are evidently influenced in our judgment by the fact that in the early hours the air is saturated with moisture, but later, owing to the rise in temperature, it is far removed rom this condition. To be accurate we must compare the masses f water vapour contained by a given volume of air at the two ifferent times. **The ratio of the mass of water vapour in a given olume of air to the mass required to saturate it at the same temperaure is called its relative humidity.** This is usually expressed as a ercentage; thus if a certain volume contains 1 gm. of the vapour, hile the amount it would contain if it were saturated is 8 gms., the lative humidity is $1/8 \times 100 = 12{\cdot}5$ per cent. Instruments used to etermine this ratio are called hygrometers.

The Chemical Hygrometer.—The relative humidity can be found rectly by the chemical hygrometer (Fig. 52). The U-tubes are led with dry calcium chloride, weighed, and connected to an pirator, which is merely a large bottle full of water with holes sed by stoppers at the top and bottom. When the water is run t air is drawn over the chloride into the bottle; as it passes rough the tubes its moisture is abstracted by the drying agent and e amount m absorbed is found by reweighing the tubes. The periment is then repeated, but the air is made to bubble through ter at the temperature of the room, so as to become saturated re it reaches the drying tubes. A furth… …ives the

The [illegible] Chemical Hygrometer

$m/m' = f/F$. Experiments with the chemical hygrometer show that this relation holds with sufficient accuracy for determinations of the relative humidity. When damp air is cooled a temperature is reached at which the moisture it contains is sufficient to pro[illegible] [illegible] saturation; any further cooling causes the water vapour to be condensed on surrounding objects in the form of dew. The temperature at which this occurs is called the dew point. The following considerations show that by determining the dew-point we can find the pressure f of the vapour in the air. Let a quantity of air in communication with the rest of the atmosphere be cooled; gas and vapour contract according to the same la[illegible] and as the joint pressure is equal to that of the atmosphere [illegible] pressure of each is unchanged. Hence the vapour pressure [illegible] ... the original uncooled air [illegible] ... [illegible] is the same

the vapour pressure at the dew-point is known from Regnat tables, since it is the maximum vapour pressure at that temperat hence f can be found.

EXAMPLE.—The temperature of the air is 16° and the dew point is 8°; the relative humidity.

From Tables we find that the maximum vapour pressures at th temperatures are 13·51 mms. and 7·99 mms. respectively; hence the relat humidity is $\frac{7 \cdot 99 \times 100}{13 \cdot 51} = 59 \cdot 1$ per cent.

Three types of dew-point instrument are described below.

Daniell's Hygrometer.—This is very similar in principle to th cryophorus. The two communicating bulbs (Fig. 53) contain ethe

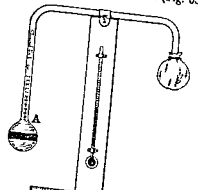


FIG. 53.—Daniell's Hygrometer.

and have been exhausted of air before being sealed. To find the dew-point all the ether is run into one bulb, A, which contains a thermometer, the second bulb is wrapped in muslin and a little ether is poured on it. The rapid evaporation from the wet material cools the bulb and the ether vapour inside it is condensed. As in the cryophorus, this results in a rapid distillation of ether from the other bulb and its temperature falls to the dew-point. In order that the deposit of moisture may be seen easily a bright band of metal is wrapped round the glass. The temperature at which dew begins to

[illegible] the enclosed thermometer, the [illegible] of the [illegible] [illegible] the dew point. The [illegible] conditions can be found as in the example given. Daniell's hygrometer is [illegible] of giving accurate results for the following reasons: (1) The enclosed thermometer gives the temperature at the middle of the liquid while evaporation takes place at the surface; (2) since glass is a bad conductor of heat (Chap. XI.) the temperatures inside and outside the bulb may differ appreciably. (3) The ether that evaporates from the outside may influence the dew-point. (4) Unless the bulb is observed through a telescope water vapour may be deposited from the observer's breath; (5) The rate of cooling cannot be controlled. These causes of error are largely removed in the two instruments described below.

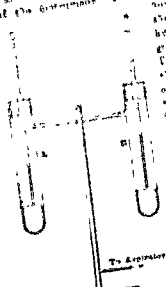


FIG. 54.—Regnault's Hygrometer.

Regnault's Hygrometer.—The lower end of a test-tube, A (Fig. 5[illegible]), is replaced by a thin silver cap in which is placed some ether surrounding the bulb of a thermometer. A piece of quill tubing passes through a cork at the upper end and extends nearly to the bottom of the tube. A side tube is connected with an aspirator placed at a distance. When water is run out a current of air is drawn through the ether in the direction of the arrows; this serves the double purpose of causing a fall of temperature through rapid evaporation, and at the same time keeps the liquid well stirred. Bubbling is continued [illegible] dew is deposited on the silver; the temperature is then [illegible] stopped. As the temperature rises again [illegible]

thermometer is read at the moment when the uew disappears. The observations are made through a telescope, and, in order that the film of moisture may be more easily detected, a second tube, B, is provided similar to the first, so that the two silver surfaces may be compared. A thermometer in the second tube gives the temperature of the air. As silver is a good conductor of heat the temperature of the ether is very little different from that of the outer surface of the cap. An additional advantage of the instrument lies in the ease with which the rate of cooling can be controlled by regulating the outflow of water from the aspirator.

Dines' Hygrometer.—This is a very simple and efficient form of apparatus; a section is shown in Fig 55. The reservoir A com-

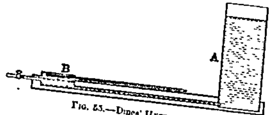


Fig. 55.—Dines' Hygrometer

municates through a tube with a shallow chamber B which has a upper and lower compartment. Into the upper division the bull of a thermometer projects, and the chamber is closed above either by thin piece of black glass or silvered mica on which the deposit of lew may easily be seen. To make an observation the reservoir is illed with a mixture of ice and water and the cold liquid is allowed o flow through the thermometer chamber until dew appears, the emperature is then taken. The flow is stopped at once and the emperature is observed at which the dew disappears. The rate of ooling can be regulated by the tap. In some forms a second eservoir is provided from which tap water is allowed to flow past e thermometer when the disappearance of the dew is being served.

Wet and Dry Bulb Hygrometer.—For many purposes the dew-point n be obtained with sufficient accuracy by means of a wet and dry lb hygrometer. This consists of two thermometers (Fig. 56).

...t the bulb of one is loosely wrapped ... immediately ...
...ch dips into a small vessel of water placed immediately. Owing to
... second thermometer gives the temperature of the air. Owing to the muslin the
...poration from the large surface exposed by the muslin the
...mperature of the wet bulb is lower than that of the other thermometer. It is easily seen that this temperature difference is connected with the humidity of the atmosphere, for if the air is dry evaporation will be rapid and the difference of temperature will be large; when no evaporation takes place the two thermometers will read alike. By comparison with one of the instruments described above a table may be constructed from which the dew-point can be found when the temperatures of the two thermometers are known.[1]

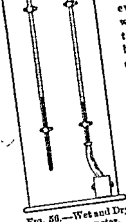


Fig. 56.—Wet and Dry Bulb Hygrometer.

1... Weight of a given Volume of Moist Air.—Since a body apparently loses weight when immersed in a fluid it will weigh more in vacuo than in air. In very accurate work all weighings must be reduced to vacuo; to do this we must calculate the weight of air displaced. Experiments with Hoffmann's apparatus show that the density of water vapour is 0·62 that of dry air at the same temperature and pressure, and the weight of a litre of dry air at N.T.P. is known to be 1·293 gms. Suppose the vapour pressure obtained from dew-point observations is f cms., the height of the barometer is H cms., the temperature $t°$ C., and that we require the weight of V litres of this moist air. The pressure of the air alone is $(H - f)$, hence its weight is (see Ex. p. 70)

$$m_1 = 1{\cdot}293V \times \frac{273}{273+t} \cdot \frac{(H-f)}{76} \text{ gms.}$$

Also the pressure of the vapour is f, hence its weight alone is

$$m_2 = 0{\cdot}62 \times 1{\cdot}293V \cdot \frac{273}{273+t} \cdot \frac{f}{76} \text{ gms.}$$

[1] ... Barton and Black, "Practical Physics," pp. 72-76.

The weight of V litres of moist air is $(m_1 + m_2)$, if therefore the volume of a body is known the mass of air it displaces can be calculated and its weight in vacuo found.

Vapour Pressure of Solutions.—The principle of the chemical hygrometer is used in measuring the vapour pressure of solutions. The experiment is conducted in the manner described on p. 98. A certain volume of air is bubbled through pure water and the vapour it contains is absorbed by calcium chloride and weighed. An equal volume is next passed through the solution, which is at the same temperature as the water, and the mass of vapour found as before. From p. 98 these masses are proportional to the vapour pressures, and as this quantity is known for water that of the solution can be calculated.

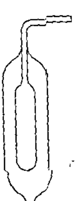


Fig. 57.

Formation of Cloud and Fog.—When the temperature of a moisture-laden atmosphere is sufficiently reduced the aqueous vapour it contains is condensed into small droplets of water forming a mist or fog. If the drops are at a high altitude they form clouds. The necessary cooling may be caused by the air expanding as it gradually rises to the upper layers of the atmosphere; it is found also that dust particles make it easier for a fog to form.

EXPERIMENT.—Replace one of the bulbs of a Looser thermoscope (p. 31) by the glass vessel shown in Fig. 57. The inner tube is connected to the thermoscope and the outer vessel to a bicycle pump. Pump in air, the compression raises the temperature and the index moves. Close the connection to the pump by a pinch-cock; the pump is now removed and the compressed air allowed to attain a steady temperature, when this is reached open the cock quickly, the gas expands and the temperature falls.

EXPERIMENT.—Shake a litre flask containing a little water so as to saturate the air. Pass through the rubber stopper two glass tubes, one connected with a bicycle pump the other closed with a pinch-cock. Compress the air with two strokes of the pump, then, after waiting a few seconds, allow it to expand suddenly by opening the cock. The air is cooled and a fog is formed. If a tube about a foot long tightly plugged with wet cotton wool is introduced between the pump and flask, so as to remove dust particles from the air which enters, the fog is largely reduced. On the other hand it is much denser if some smoke from burning paper is first introduced. It is doubtless due to this effect of dust particles that fogs are so common in large towns.

CHAPTER X

FIRST LAW OF THERMODYNAMICS. MECHANICAL EQUIVALENT OF HEAT

In the preceding pages we have frequently supposed that there is some connection between the heat contained by a mass of gas and the kinetic energy of its molecules. It has also been found necessary in the case of an expanding gas to assume some relation between heat and work to account for the difference between the specific heat at constant pressure and that at constant volume (p. 35). It will be shown in this chapter that heat is a form of energy and that other forms of energy may be converted into heat.

Units.—In the centimetre-gram-second (*C.G.S.*) *system of units* the unit of force is the dyne; it is that force which, acting on 1 gm. for 1 sec., gives to it a velocity of 1 cm. a second. In the English system the unit is that force which, acting for 1 sec., imparts to 1 lb. of matter a velocity of 1 ft. per second; it is called the poundal. When a force F moves a body through a distance s, measured parallel to the direction in which the force acts, Fs units of work are expended. If F is in dynes and s in cms. the work is given in ergs; when F is in poundals and s in feet the work is expressed in foot-poundals. The weight of 1 lb. is sometimes used as the unit of force. It is shown in books on mechanics that 1 lb. $= g$ poundals, where g is the acceleration due to gravity. When a force of 1 lb. moves its point of application through 1 ft. in a direction parallel to the force 1 ft.-lb. of work is done. Work is expended against the force when the motion is in the opposite direction to that in which the force acts. Thus if a flywheel of radius R cms. is forced round in opposition to a frictional force of F dynes applied to its rim, during each revolution a point on the rim is moved through a distance 2πR cms., and the work done is F . 2πR ergs. Let two equal and opposite forces F be applied to the ends of a lever of length d and at right-angles to it. During

revolution each force does $2\pi \frac{d}{2}$. F units of work, and the work done by the couple is $Fd \cdot 2\pi$, i.e. it is equal to the product of the moment of the couple and the angle in radians through which the arm is turned.

The calorie has already been defined, but another unit of heat is sometimes used; it is the amount of heat required to raise 1 lb. of water through 1°, either Fahr. or Centigrade; this is called the lb.-degree unit.

Experiments showing that Heat is a Form of Energy.—Numerous experiments show that heat may be generated by the expenditure of work. Thus a hundred years ago Davy showed that two pieces of ice could be melted by rubbing them together, the heat generated by moving them against the frictional forces was sufficient to cause melting. Similarly Count Rumford observed that during the process of boring a cannon from a solid block of metal sufficient heat was generated to boil a large quantity of water. The amount of heat gained was conditioned entirely by the amount of work expended in driving the drill. The method used by some savage tribes to obtain a fire is a parallel case, a blunt wooden point is caused to rotate rapidly in a shallow hole cut in a block of wood, enough heat is thus produced to kindle a flame. A block of metal is appreciably warmed by hammering, and the lower end of a bicycle pump is heated on account of the work expended in compressing the air.

EXPERIMENT.—*Compress the air in the experiment with a Looser's thermoscope, p. 103; notice that its temperature rises. This is an instance of nearly adiabatic compression (p. 113).*

Mechanical Equivalent of Heat.—The first experiments to find the numerical relation between the work done and the heat produced are due to Joule; the object was to expend a known amount of work in the production of heat and to measure the heat developed. The results showed that no matter how the work was done, the ratio of work done to the heat generated was constant. This is the first law of thermodynamics. In symbols, if W is the work expended in the production of H units of heat then $W/H = J$, or $W = HJ$, where J is a constant called the mechanical equivalent of heat. Modern experiments show that $J = 4{\cdot}18 \times 10^7$ if W is measured in ergs and H in calories. The equation therefore tells us that to generate one calorie $(H = 1)$ $4{\cdot}18 \times 10^7$ ergs of work must be done.

Joule's Experiments.—In these experiments work was expended churning water contained in a calorimeter and the resulting mperature rise θ was measured If M was the total water equivaat in grams of the calorimeter and its contents the heat generated as $M\theta$ calories. The apparatus shown in Fig. 58 was used to easure the work expended The water was churned by a paddle rrying a number of vanes, these passed between a system of fixed anes attached to the walls of the calorimeter (see figure below). o prevent conduction of heat from the vessel as its temperature

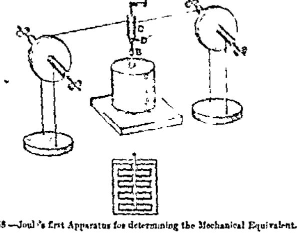


FIG. 58 —Joule's first Apparatus for determining the Mechanical Equivalent.

rose, the metal axis of the paddle was interrupted at B by a boxwood cylinder. A flexible cord passed round the wooden drum C and its ends were wound on to two large pulleys supported on friction wheels. The pulleys carried equal weights, which were supported by strings wound round the axles, their height from the ground could be read off vertical scales. When the weights were allowed to fall they made the pulleys revolve and the paddle was put in motion. The pin D was then quickly removed, when the weights could be wound up again by a handle without turning the paddle. The fall experiment was repeated a large number of times and the temperature of the water was read at frequent intervals. Let m be the mass

9. [illegible] corrections must be [illegible] of which the following are the chief

(1) As the calorimeter is at a higher temperature than its surroundings it will lose heat by radiation and conduction [illegible] as the experiment lasts. The observed temperature will therefore be too small. The necessary cor[illegible] can be found by noting the rate of cooling at the beginning [illegible] of the other observations. Conduction losses are [illegible] the vessel on a badly conducting base.

(2) The weights are moving with a velocity v cm/sec. when they reach the ground, and each has a kinetic energy $\frac{1}{2}mv^2$ ergs which must be subtracted from their original potential energy to [illegible] work done in turning the pulleys and paddle. Hence the work in n falls is $2n(mgh - \frac{1}{2}mv^2)$. As it was found that the weights [illegible] with a uniform velocity before they reached the ground, v could be observed by noting the time taken to move over a measured di[illegible] near the end of their path.

(3) A certain amount of work is spent in overcoming fric[illegible] the moving parts outside the calorimeter. To determine the fric[illegible] force the drum C was disconnected from the paddle at D and the [illegible] from the pulleys was passed round it in such a manner that [illegible] one weight fell it caused the other to rise. A mass m_1 gra[illegible] placed on one weight to make it fall, this additional mass bei[illegible] chosen that the motion was uniform. The frictional resistance [illegible] therefore m_1g dynes and the total work done against it was n[illegible] ergs. Hence the total work expended in churning the water [illegible]

$$\{2n(mgh - \tfrac{1}{2}mv^2) - nm_1gh\}\ \text{ergs}$$

Actually Joule took as the unit of work the ft.-lb. and for [illegible] unit the quantity of heat required to change the temper[illegible] 1 lb. of water by 1° Fahr. With these units J was found to [illegible]2. In other experiments he used an iron paddle to stir merc[illegible] an iron vessel; he also measured the work spent in compres[illegible]

ir into a reservoir immersed in a calorimeter. The value found or the mechanical equivalent was practically the same in every ase

Rowland's Experiments.—Considering the small rise of temperaure obtained, which was about half a degree, Joule's results are urprisingly consistent, but the most accurate experiments by the method of churning water are those of Prof. Rowland. In these the temperature rose at the rate of 0·5° per minute. The calorimeter and stirrer were similar to Joule's except that the paddle projected through the base and was turned by a steam engine (Fig. 59). The

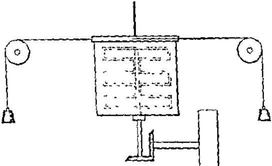


FIG. 59 —Diagrammatic Sketch of Rowland's Apparatus.

p of the calorimeter was fastened to a circular wooden disc which ung from the end of a thin wire. When the paddle turned the ction of the water tended to move the calorimeter in the same rection, but its motion was prevented by passing a string round he disc and hanging equal weights from the ends. If d is the iameter of the disc and m the mass of one weight, the moment of he couple which stops the motion is mgd. Now the water exerts qual and opposite couples on the calorimeter and the paddle, hence he moment against which the latter is forced round is $mg \cdot d$, and he work done in one revolution is $2\pi \cdot mgd$ (p. 106). The work xpended during n revolutions is therefore known, and as the heat eveloped can be measured, J can be found as before. The radiation rrors are relatively much smaller than in Joule's experiments.

A Laboratory Method of determining the Mechanical Equivalent.—A simple apparatus can be used for this purpose. An outer brass cone, A, shown in section in Fig. 60, is fixed in ebonite to the base of a brass cylinder and is held in position by a ring of ebonite near the top (this substance is a bad conductor). A second brass cone, B, fits smoothly in the first and is attached at its upper end to a circular wooden disc of diameter d. The inner cone contains a known mass of water, a stirrer, and thermometer. By means of an endless band going to a small motor the outer cone is rapidly rotated; the friction

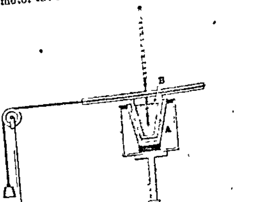


Fig. 60.—Laboratory Method of finding the Mechanical Equivalent.

between the two tends to make the inner cone follow in the direction, but this is prevented by a weight of m gms. fastened string passing round the wooden disc. By suitably adjusting speed and the weight the latter can be kept floating in the a as the couples on the cones are equal and opposite the motion cone is opposed by a couple whose moment is $mg\frac{d}{2}$, and the work done in n revolutions is $mg\frac{d}{2}.2\pi n$ ergs. Knowing the equivalent of the two cones and their contents, the heat ped by the friction in n revolutions can readily be found and calculated.

Work done by a Gas expanding against a Uniform Pressure.—Let a quantity of gas be confined in a cylinder which is closed by a light piston of area S, and suppose the gas to expand, pushing the piston out a distance x cms. against the atmospheric pressure of p dynes. The increase in volume of the gas is xS cms.^{3} Also the total external pressure on the piston is pS dynes, and the work done during the expansion is $p\mathrm{S} \,.\, x$ ergs (force × displacement); *i.e.* the work $= p \,.\, \delta v$, where δv is the increase in volume. Let us make use of this result to calculate the work done against the atmospheric pressure when a gram of water at 100° is converted into steam. If the barometer stands at 76 cms. it is known that the increase of volume is 1690 cms.^{3} approximately, and this expansion takes place against the atmospheric pressure. The density of mercury being 13·6, the atmospheric pressure in dynes/cm.^{2} is $13{\cdot}6 \times 76 \times 980 = 1{,}013{,}000$, hence the work done $= 1{,}013{,}000 \times 1690$ ergs. The equivalent of this in calories, taking $\mathrm{J} = 4{\cdot}2 \times 10^7$, is $\dfrac{1{,}013{,}000 \times 1690}{4{\cdot}2 \times 10^7} = 40{\cdot}7$ cals.

This accounts for part of the latent heat of vaporisation, the remainder is spent in pulling the molecules of water apart against their mutual attraction.

Calculation of J from the Two Specific Heats of Air.—Let a gram of air at a pressure p_1 dynes and absolute temperature T_1° occupy v_1 cms.^{3}. To raise its temperature 1° requires C_v cals. if the volume is kept constant, C_v being the specific heat at constant volume. On other hand, if the pressure is constant the gas expands to a new ume v_2, and work equal to $p_1(v_2 - v_1)$ ergs is done on account of s expansion; an amount of heat C_p cals. must be supplied in s case, C_p being the specific heat at constant pressure. The ference $(\mathrm{C}_p - \mathrm{C}_v)$ cals. is used to provide the work done in panding; multiplying by J to bring this to ergs we have two pressions for the work done, and these must be equal, hence

$$p_1(v_2 - v_1) = \mathrm{J}(\mathrm{C}_p - \mathrm{C}_v)$$

ut from the gas equation (p. 69)

$$p_1v_1 = \mathrm{RT}_1 \text{ and } p_1v_2 = \mathrm{R}(\mathrm{T}_1 + 1)$$

ence $\quad p_1(v_2 - v_1) = \mathrm{R}$ and $\mathrm{J}(\mathrm{C}_p - \mathrm{C}_v) = \mathrm{R}$

ut $\mathrm{R} = p_1v_1/\mathrm{T}_1 = p_0v_0/273$, where p_0, v_0, are the pressure a f 1 gm. of air at 0° C. or 273° absolute. From measureme

ensity of air it is known that 1 cm.³ at N.T.P. weighs 0·00129 gm
ence the volume of 1 gm. under these conditions is = 1,000
cms.³ = v_0. Also p_0 = 1,013,000 dynes,

$$R = \frac{1{,}013{,}000}{273 \times 0{\cdot}00129}$$

Now $C_p = 0{\cdot}2375$ cals. and $C_v = 0{\cdot}169$ cals.; substituting these three values in the equation $R = J(C_p - C_v)$ we get $J = 41{\cdot}4 \times 10^6$.

In making this calculation we have assumed that all the work done by the gas in expanding is spent against the external pressure. It is, however, possible that some work is done in pulling the molecules apart against their mutual attractions, just as work has to be done when a weight is raised from the ground against the attraction of the earth. Joule was the first to show that this effect was negligible; his apparatus is shown in Fig. 61. A metal reservoir, A, was filled with dry air at a pressure of 22 atmospheres; another reservoir, B, was exhausted and joined to the first through a tube furnished with a stop-cock. The two were placed in a calorimeter containing water an when the temperature had becom steady the stop-cock was opene

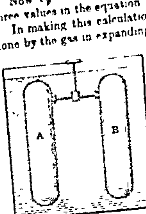


FIG. 61.—Joule's Apparatus to show that the Internal Work done by an Expanding Gas is Zero.

No work was done against the external pressure since B w exhausted, but if the molecules exercised an appreciable attracti on each other work would be spent in increasing their distan apart. This internal work would use up some of the heat energy the gas and the temperature would fall. As it was found that temperature did not change appreciably Joule concluded that internal work was done; the calculation just given is theref justifiable. Later experiments by Joule and Thomson, in whic more delicate method was used, have shown that this conclu is not strictly true. These experiments we shall not attemp scribe.

Methods of finding J. Conservation of Energy.—Se ds other than those already given have been used to dete

mechanical equivalent. Thus when an electric current passes along a wire heat is generated, the energy in ergs can be measured electrically, and hence J can be found; this method is described on p. 406 The nett result of all such experiments is to show that energy in all its forms, chemical, mechanical, potential, or electrical, may be converted into heat, and to generate one calorie requires the expenditure of $4{\cdot}18 \times 10^7$ ergs. We may regard heat as a kind of common denominator to which all other forms of energy can be reduced. Also experiment shows that the various forms of energy are interchangeable; thus the potential energy of water at the top of the Zambesi Falls is convertible into the kinetic energy of a water ...ne, and this is made to drive a dynamo which generates electrical energy; the chemical potential energy in coal is transformed into mechanical energy in the steam engine and so on. In all cases, as ...s experiment goes, it is found that no energy is lost, it merely changes its form. This statement is called the law of conservation of energy; it is one of the most important discoveries of modern ...ce.

...othermal and Adiabatic Changes.—Any variation in the state of a system which takes place at constant temperature is called an isothermal change. The fusion of a solid at its melting point and the ...isation of a liquid at its boiling point are instances of such ...rly Boyle's law gives us the isothermal relation between the ...re and volume of a perfect gas. An adiabatic change is one ...akes place without heat entering or leaving the system in ...on. Thus the expansion of a gas as in the experiment on p. 103 ...diabatic expansion because it takes place so quickly that heat ... flow into the gas from the surroundings while the change is proceeding. As work is done in this expansion and no ... supplied, a portion of the heat energy of the gas is converted ...rk and the temperature falls. The converse happens when ...uddenly compressed in a bicycle pump, work is done on the ..., as no heat leaves it, its energy is increased; this appears as ...n temperature. It can be shown that if the pressure and ... of a gas are changed adiabatically from p_1, v_1 to p_2, v_2, ...vely, these quantities are connected by the equation ...$p_2v_2^\gamma$, where $\gamma = C_p/C_v$, the ratio of the specific heats. This ...diabatic relation corresponding to the isothermal one given ...Boyle's law equation. We shall meet with adiabatic changes ...ction on sound.

EXAMPLE.—A quantity of air at 76 cms. pressure is suddenly compressed to half its volume, calculate the new pressure.

Here
$$v_1/v_2 = 2 \quad \text{and} \quad \gamma = 1.4$$
$$p_2 = p_1\left(\frac{v_1}{v_2}\right)^\gamma = 76\,(2)^{1.4}$$
$$\therefore \log p_2 = \log 76 + 1.4 \log 2$$

From a table of logarithms we find $\log p_2 = 2.3022$ and $p_2 = 200.5$ cms.

Had the compression taken place isothermally the final pressure would have been 152 cms.; we see then that the resistance to an adiabatic change is greater than it is to one which takes place at constant temperature.

EXAMPLES ON CHAPTER X

1. An engine consumes 40 lbs. of coal of such calorific power that the hea developed by the combustion of 1 lb. is capable of converting 16 lbs. of wate at 100° into steam at the same temperature, and during the process the engi performs 16×10^6 ft. lbs. of work. What percentage of the heat produced wasted? [Lat. ht. of steam = 536.] (L. '81.)

2. Describe Joule's method of determining the mechanical equivalent heat from the expansion of compressed air. Explain what happens when is allowed to expand into a vacuum. (L. '82.)

3. Distinguish between the specific heat of air at constant pressure and constant volume, and show how to determine the latter when the former, toget with the mechanical equivalent of heat, are known. (L. '83.)

4. One gm. of air is heated under constant pressure from 0° to 10°, deter the work in ergs and in gms.-cms. due to the expansion. [Coefficien expansion = 1/273, 1 c.c. of air at N.T.P. = 0·001293 gms., 1 c.c. of mercu 0° = 13·596 gm., g = 981 cm. secs. (L. '84.)

5. When temperatures are expressed on the Centigrade scale the latent of fusion of ice is represented by 80, and the mechanical equivalent of he 423·9 (metre-gms.). Express the same quantities on the Fahr. scale and ex why one is represented by a larger and the other by a smaller number. (L

6. Water oozes slowly from under a pressure of 20 atmospheres and lected in a vessel. How much hotter is this water than it was inside? (I

7. A quantity of damp air under pressure is suddenly allowed to e Describe what happens, and show what has become of the energy of th pressed air. (L. '91.)

8. A lb. of coal in burning can raise 8000 lbs. of water 1°. Used in a the coal supplies 1,400,000 ft.-lbs. of work per lb. burnt. What fractio heat is transformed to work? [Mech. equiv. = 1400 ft.-lbs.-deg. Cent.]

9. Water at 15° C. and 1000 atmospheres pressure is passed through plug and escapes at 1 atmosphere pressure. Calculate the temperatu escaping water, given 1 atmosphere = 10^6 dynes per cm.², and the m equivalent of heat = 4.2×10^7 ergs.

CHAPTER XI

ROPAGATION OF HEAT. CONDUCTION AND CONVECTION

luction, Convection, Radiation.—Heat travels from one point nother by three processes named respectively (1) Conduction, Convection, (3) Radiation. As a typical instance of the first nay take the case of an iron bar heated at one end. According ie kinetic theory of matter the molecules of a substance are osed to be oscillating to and fro, the motion becoming more ous as the temperature rises. Owing to collisions the molecules hot end share their energy with their slower moving neighbours, in turn carry energy to the next layer, and so a rise of temperaravels down the bar, although the molecules themselves do not from their mean positions. In the process of convection the l particles wander through the substance carrying their heat them, and by frequent collisions the rise in temperature is d throughout the whole mass. Convection currents can only in liquids and gases.

ERIMENT.—Fill a large beaker with cold water and drop down the of it a single crystal of potassium permanganate. Heat the beaker tely under the crystal by a small flame, convection currents can be seen rising up the central portions and returning by the colder sides. ERIMENT.—Make a complete rectangle out of glass quill tubing, fill it ter and drop in a crystal of potassium permanganate. Hold it with vertical and gently heat one side; a convection current rises from the art and travels round the tube

ach of these processes heat is propagated through the interof particles of matter; in the third process, radiati through space from which all matter is r , at a height of a few hundred miles the must be practically nil, yet heat millions of miles of this vac of a hot body emit

[illegible]ergy . . . when these fall on [illegible] part of the energy they carry is [illegible] the [illegible] of the [illegible] of the cold body to [illegible] [illegible] As we cannot [illegible] the [illegible] is assumed that the universe is [illegible] through which [illegible] of the other, concerning the properties of [illegible] with [illegible] for transmitting waves, we know [illegible] distance between successive waves, [illegible] It is known that the distance [illegible] only a fraction of a mil[illegible] the wave-length as it is called, is very small, only a fraction of a mil[illegible] [Chap]ter XXIII for an exact definition of wave-length). The effects produced by the waves are dependent on the wave-length, the longer ones produce heating effects, others produce the sensation of light when they fall on the retina of the eye, while the shortest are chiefly notable for the chemical changes they promote. Heat propagated in this manner is sometimes called "radiant heat," but the term is not a good one, for the energy does not appear as heat except when the waves fall on matter; in addition light waves produce heat, and the only difference between the two is in their wave-length. A better term is radiant energy or merely radiation. Owing to the similarities just mentioned the study of radiation will be deferred for the most part until we come to the section on light.

EXPERIMENT.—Cover the bulb of a thermometer with soot by holding it near a smoky flame. Place it at the centre of a small glass flask closed by a rubber stopper, and pump out as far as possible all the air. No matter how far the exhaustion is pushed the thermometer rises in temperature when held near a hot body, showing that radiation can travel across from the flask w[illegible] without the aid of matter.

Examples of Heat Conduction.—Silver, copper, and metals gener[ally] are good conductors of heat; powders, liquids, and gases are [poor] conductors. Thus on a frosty morning a metal door-knob [feels] colder than the wood because it rapidly conducts heat away [from] the hand, but if the door is exposed to a hot sun the metal part [feels] hotter than the wood because it conducts more heat to the h[and]. If a piece of thin paper is wrapped round a brass rod it may be [held] in a flame for a few seconds without being scorched; if the br[ass] replaced by wood it is scorched at once because the latter subs[tance] does not conduct heat away with sufficient rapidity. A sphere [of] solder can be melted in a small paper bag and water can be boi[led] in a similar receptacle; in the last instance the heat is carried aw[ay] by convection currents. It is owing to bad conduction that a ligh[ted] [m]atch can be held in the hand and glass tube worked in a blow-p[ipe]

me; for the same reason the lower end of a lighted candle is i
lted.

EXPERIMENT.—Replace the bulbs of a Looser thermoscope (Fig 17) bottomed flasks with the flat parts uppermost. On one flask place a opper, on the other an equal disc of iron. When a flask of boiling w aced on each the thermoscope shows that the most heat passes thro opper. If a disc is replaced by a shallow, hollow vessel filled with w little heat passes through; liquids are very bad conductors. This is ngly shown by the next experiment.

XPERIMENT.—Attach a small lump of ice to a sinker and drop it to the om of a test-tube nearly filled with water. The tube may now be held inclined position and heated near its upper end until the water boils, but ient heat is not conducted downwards to melt the ice. If the tube had heated from below convection currents would have equalised the tem- ure throughout the mass.

lases also are bad conductors; woollen clothing is warmer than n largely because of the air it entangles, convection currents et up with difficulty among the fibres of the material, hence heat ot get through except by conduction. The feathers of birds lown quilts owe their efficacy to a similar cause.

XPERIMENT.—*Leidenfrost's phenomenon.* Heat a clean sheet of metal t s and let a few drops of water fall on it. They run to and fro over th like mercury on clean glass, but do not boil away furiously as we migh . At the first contact with the plate a cushion of vapour is formed whic ts heat reaching the liquid except by conduction through this layer o ation. With care it is possible to pass a beam of light between th and the plate. Remove the flame; as the plate cools the cushion becomes unable to support the drop, contact with the plate follows an uid boils away rapidly. Owing to the bad conduction of a layer it is possible to lift a piece of red-hot coal with the fingers withou provided the hand is first thoroughly wetted.

XPERIMENT.—Lower a piece of fine copper gauze into a Bunsen flame ne appears to be pushed down and does not get to the upper side te unless it is very hot.

XPERIMENT.—Fix the gauze in position a couple of inches above the t urner before the tap is turned on. If the gas is now lighted above t a flame is unable to penetrate below.

order that gas may be ignited it must be raised to a certa um temperature, but the metal conducts heat away
is temperature is not reached above the
ent or below it in the other. This
ty lamp used by miners. It somet
an explosive mixture

ith a naked flame an explosion would follow. To hinder this the ame is entirely surrounded by a mantle of copper gauze, then, owing o conduction, the temperature outside never becomes high enough o ignite the mixture except in extreme cases.

Instances of Heat Convection.—When a building is heated by hot water a boiler is placed in the basement and pipes slightly inclined to the horizontal go from this to a cistern in the top storey. The hot water in the boiler has a smaller density than that in the pipes, it therefore rises, carrying heat with it. As it passes through the various rooms its heat is radiated from the surfaces of the pipes, and by the time it reaches the cistern it is cool. From here it sinks through vertical pipes to the boiler again and a continuous circulation is thus brought about by convection. The draught of a chimney is due to convection currents of hot air. On a hot summer's day land near the sea is at a higher temperature than the water, owin to the larger specific heat of the latter. An upward current of ho air is produced over the land which is replaced by a colder one comi off the sea, thus causing a sea-breeze. In the evening the land coo more quickly and the conditions are reversed; the prevailing bree is then from land to sea. Joule's apparatus (p. 55) depends for i action on convection currents.

Temperature at any Point in a Bar.—When a cylindrical bar heated at one end some time elapses, it may be several hours, bef the temperature at every point becomes steady. Let us consi what conditions influence the temperature of a small slice of the not far away from the hot end before and after this steady stat established. If the substance is a good conductor much heat travel to the slice; of this a part flows away across the colder a further amount is lost by radiation from the curved surface, part is retained, in the earlier stages, to raise its temperature. change in temperature will be great in proportion as the the capacity (mass × specific heat) is small. After some time the perature becomes steady at every point. When this state is re thermal capacity has no further influence, the heat a slice recei its hot end now either flows away by conduction or is lost from the curved surface. The latter losses are to surface emission. If we have two equal bars of neg for which we may regard the surface losses as neg of the temperatures at two corresponding sectio

depend partly on the conducting powers and partly on the thermal capacities until the steady state is reached; afterwards it will depend on conduction alone, the better the conductor the higher the temperature. If heat is lost from the surface the temperature will be reduced at every point because there is less heat to be transmitted.

EXPERIMENT.—Take equal wires of copper and bismuth, coat them with paraffin wax and put one end of each in a Bunsen flame. The wax melts more quickly along the bismuth in the early stages, but in the end more is melted on the copper. The latter metal is therefore the better conductor, but the small thermal capacity of bismuth more than compensates for this while the temperature is rising. It can be shown that the ratio of the thermal conductivities, as defined in the next paragraph, is equal to the ratio of the squares of the lengths along which the wax is melted.

These remarks show that in comparing conducting powers we must heat the bars long enough for the steady state to be reached, otherwise the results depend on the thermal capacity. It simplifies matters also if the surface emission can be neglected. Now, if a thick bar is split up into two others of half the section more surface is exposed, hence the surface losses are of less importance in thick bars.

Thermal Conductivity. Searle's Apparatus.—We must now define more exactly the conducting power, or, as it will be called in future, the thermal conductivity of a substance. Consider a plate of the substance of thickness l cms., whose opposite faces are kept at temperatures θ_1 and θ_2. Heat will flow from the hotter to the colder side, and if we consider an area S some distance away from the edges the lines of flow will be perpendicular to the faces. The antity of heat that flows across this area in t seconds can be shown be—

(1) Proportional to the area S.

(2) Proportional to the time t.

(3) Proportional to the difference of temperature $(\theta_1 - \theta_2)$ etween the faces.

(4) Inversely proportional to the thickness l.

If we denote the quantity of heat in calories by Q,

hen
$$Q \propto S\frac{(\theta_1 - \theta_2)}{l} \cdot t$$

r
$$Q = kS\frac{(\theta_1 - \theta_2)}{l} \cdot t \text{ cals.}$$

where k is a constant called the thermal conductivity of the material

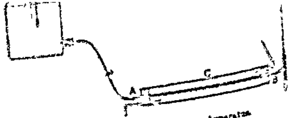


FIG. 83.—Simple Conductivity Apparatus.

water may be brought into contact with the heated wall a heat pe[...]
is fixed along the axis of AB; this throws the liquid into eddies an[...]
well mixed.

EXAMPLES ON CHAPTER XI

1. Define conductivity for heat and show how the fundament[...] length, mass, and time enter into its numerical specification. [...] conductivity of iron as 0.17 C.G.S. units, what difference of temper[...] exist between the surfaces of an iron wall, 3 cms. thick, through e[...] metre of which heat is streaming, from a furnace on one side to b[...] other, at the rate of 30,000 C.G.S. units per minute? (L[...]

Radiation has long been falling on a slab with a blackened [...]

sq. decimetre of which absorbs 10,000 ergs per second; and the energy is transmitted to a back surface, 0·5 cm. distant, where it is removed by water. What steady difference of temperature must exist between the two surfaces of the slab if its conductivity is 0·02 C.G.S. units? (L. '92.)

3. A metal vessel, 1 sq. metre in area, and whose sides are 0·5 cm. thick, is filled with melting ice, and is kept surrounded by water at 100°. How much ice will be melted in an hour? The conductivity of the metal is 0·02 and the latent heat of fusion of ice is 80. (L. '95.)

4. Suppose 10 cms. of ice to have already formed on a pond, and that the air is at $-5°$. How long approximately will it take for the next mm. to form? [Conductivity of ice = 0·005, latent heat = 80.] (L. '04.)

CHAPTER XII

PROPAGATION OF HEAT. RADIATION

Instruments used.—It has been seen that when radiation falls on a body part of it is absorbed, causing a rise in temperature. Any apparatus whose condition is appreciably changed by the reception of a small quantity of heat can therefore be used as a detector of radiation. No substance is known which absorbs all the radiation which falls upon it, but lamp-black, or soot, absorbs more than 90 per cent., and, what is more important, it absorbs all radiations equally no matter what their source. A differential air thermometer wit one bulb covered with lamp-black was used as a detector by th early experimenters, but it is now superseded by electrical method The principles on which these are based will not be fully understoo without some knowledge of electricity, but it may be briefly stated that when two dissimilar metal rods are joined together at their end and one junction is heated, an electrical current flows through them This can be measured by a galvanometer. A set of such antimony bismuth junctions are covered with lamp-black and arranged s that the effects of the separate junctions are added, such an arrang ment is called a thermopile. It is usually placed inside a metal co to screen it from all radiations except those coming in a defin direction, when these fall on the junctions they are heated and current passes through the galvanometer (see Chap. XL.). Anot detector consists of a thin strip of platinum covered with lamp-bla When its temperature rises, owing to incident radiation, its electr resistance is increased; this is measured by suitable means suc a Wheatstone's bridge (p. 382). An apparatus of this type is c bolometer.

Emissive Power.—The rate at which a body loses heat by r may depend (1) On the nature of the surface; (2) O ture of the body and of its surroundings; (3) O

material of which the body is composed. When radiation coming through air falls on a surface some of the waves may merely be turned back or reflected; this is especially the case when the surface is bright. A good reflector is therefore a bad absorber. But reflexion takes place equally when the waves are travelling from the interior of the substance towards the air; hence, if the surface is a ...od reflector, most of the heat is returned to the interior. Thus ...od reflectors emit very little radiation.

EXAMPLE.—A bright kettle takes longer to heat but retains its heat better ...an a black one.

A substance which absorbs all the radiation which falls upon it ... called a "perfectly black" body. Practically we may treat lamp-black as such. The ratio of the quantity of radiation emitted per sec.

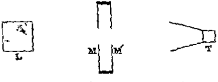


FIG. 64.—Apparatus for the Comparison of Emissive Powers.

by a cm ^{2} of a surface to the quantity emitted by a cm ^{2} of a perfectly black body under equal conditions is called the emissive power of the surface. Emissive powers can be compared by the method of la Provostaye and Desains. A metal cube, L (Fig. 64), usually called a Leslie's cube, is filled with boiling water or other liquid and its vertical faces are covered with the substances to be compared. About 50 cms. away is a thermopile T (the galvanometer used with this is not shown), and between this and the cube is a double metal ...een MM'. The sides of the screen facing the cube and thermopile ... covered with lamp-black while the inner faces are bright. If ... left face of M were bright it would be possible for radiation ...ling on it to be reflected back to the cube and from thence to the ...ermopile; the bright face behind hinders direct radiation from M ... the thermopile, while M' acts as an additional check to this and so prevents the reflection of radiation coming from the right. ...he emissive powers are proportional to the currents produced. It ... found that a lamp-black surface is the best radiator, but bright

[illegible]

Newton's Law of Cooling.—[illegible] the rate at which [illegible] part of the [illegible] through the [illegible] are less im[illegible] [illegible] large [illegible] by covering th[illegible]. This is arranged for by covering the radiating surface with lamp-black.

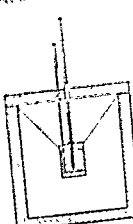


Fig. 63.—Apparatus to test Newton's Law of Cooling

EXPERIMENT.—Take a small, thin-walled calorimeter which can be closed with a met[illegible] lid provided with a hole for a stirrer and the thermometer; cover it with lamp-black by holding it on a smoky flame, then fill with water at temperature about 50°. Hang it in a d[ouble] walled vessel (Fig. 63), the space between the double walls being filled with water in which a thermometer is placed. (Instead of the [illegible] thermometer and [illegible] put a heavy weight in a calorimeter and [illegible] it in a large beaker of water.) Keep the w[ater] [illegible] water well stirred and note its temper[ature] every half minute. Plot a curve showing the difference of temperature between the i[nner] and outer vessels at different times. [illegible] this read off the fall of temperature d[illegible] any minute, i.e. the rate of cooling, and [illegible] the temperature at the middle of each m[inute] [illegible] the temperature rate of cooling and the [illegible] This [illegible]

Tabulate these results and plot a new curve showing rate of cooling and the excess of temperature of the hot water over that of its surroundings. This is found to be a straight line if the temperature excess is not more than [illegible] degrees, hence within these limits the rate of cooling of a body is proportional to its excess of temperature over that of its surroundings. This is called Newton's law of cooling although Newton's experiments were carried out under very different conditions.

For large temperature differences the law does not hold, as the curve shows. It holds sufficiently well, even when convection currents are present, to enable us to calculate the radiation losses from the vessels used in calorimetry; in these cases the excess of temperature is usually small. Within the limits in which Newton's law is true the rate of cooling is independent of the actual temperatures of the hot body and its surroundings; it depends only on the *difference* of temperature between the two. T[illegible]

he rate of cooling when the small calorimeter is at 50° and the uter vessel at 45° is the same as if the respective temperatures vere 20° and 15°.

Effect of the Nature of the Liquid on the Rate of Cooling.—If the mall calorimeter of the last experiment is filled with turpentine nstead of water it is found that the rate of cooling is faster although he blackened surface has remained unaltered. The results take a very simple form if instead of comparing the rates of cooling we ompare the amounts of heat lost. To make the comparison the specific heat of turpentine must be known. Suppose with water in the calorimeter it takes t_1 seconds for the temperature to fall from 25° to 20°. If m is the mass of the calorimeter, s its specific heat, and m_1 the mass of water contained, the heat lost is $5(m_1 + ms)$ cals., and the heat lost per second is $5(m_1 + ms)/t_1$. Repeat the experiment through the same interval of temperature when the water is replaced by turpentine. Let m_2 be the mass of turpentine, s_2 its specific heat, and t_2 the time required. Then the heat lost per second is $5(m_2s_2 + ms)/t_2$ cals. It will be found that *the heat lost per second is the same in each case*, hence

$$\frac{m_1 + ms}{t_1} = \frac{m_2s_2 + ms}{t_2}$$

If the surface is unaltered the heat lost per second is independent of the nature of the liquid.

Specific Heat from the Rate of Cooling.—The principle just given can be used to find the specific heat of a liquid. Using the apparatus shown in Fig. 65 the time taken to cool from, say, 35° to 30° is observed, first with water in the small calorimeter, next when it contains an equal volume of the liquid. (Equality of volume ensures that the cooling surfaces will be equal in the two experiments.) The specific heat is calculated from the equation just given, where m_2 and s_2 refer to the liquid. Regnault found that the method was useless for powders or solid bodies. The reason for this is obvious; the rate of cooling depends on the rapidity with which heat is conducted from the interior to the surface, i.e. upon the thermal conductivity. For liquids it is very convenient, especially when but a small quantity of the substance is available; thermal conductivity does not enter in this case since the liquid is well stirred. In order that the surface may not be altered it is well to heat the liquids to

a suitable temperature in a beaker before placing them in the calorimeter.

EXPERIMENT.—Compare the emissive powers of bright and black surfaces by noting the rate of cooling of a bright calorimeter containing water, then smoke its surface and observe the rate of cooling over the same temperature interval as before. The emissive powers are inversely proportional to the times of cooling.

EXPERIMENT.—*Calculate the heat emitted in 1 sec. from a sq. cm. of surface when its mean temperature is 25°.* The time taken for the calorimeter in the last experiment to cool from 27° to 23° is found, the heat emitted per sec. is then $4(m_1 + ms)/t_1$ cals. This must be divided by the area of the calorimeter surface. The heat emitted in 1 sec per cm.² when the body is 1° hotter than its surroundings is sometimes called the surface emissivity. Calculate this for the calorimeter surface.

Absorption of Radiation.—Let a quantity of energy equal to Q ergs fall on a surface in a second and let Q′ be the amount absorbed; the ratio Q′/Q is called the absorptive power, or coefficient of absorption, of the surface. It is difficult to measure absorptive powers directly, but they may easily be compared by a method due to la Provostaye and Desains. A thermometer bulb coated with one of the substances in question is placed in a closed box and the radiation allowed to fall on it through a suitable lens. The temperature rises until the heat lost by radiation is equal to that gained by absorption; let the steady temperature be $t_1°$. A cooling curve is now plotted for the thermometer starting at a temperature slightly higher than $t_1°$; from this we can determine as on p. 126 the rate of cooling when the mean temperature is $t_1°$, let it be $\theta_1°$ per second. Then if M is the thermal capacity of the bulb the heat lost per second is $M\theta_1$. But if Q is the radiation falling on it per second when exposed to the source and A_1 the absorbing power, the heat absorbed in a second is A_1Q, hence

$$A_1Q = M\theta_1$$

Similarly for a second substance exposed to the same source

$$A_2Q = M\theta_2$$

hence

$$\frac{A_1}{A_2} = \frac{\theta_1}{\theta_2}$$

The results show that substances with large emissive powers have large absorptive powers. By means of Ritchie's apparatus

it may in fact be proved that the emissive power of a surface, as defined on p. 125, is equal to its absorptive power. The bulbs of a differential air thermometer are formed from cylindrical metal boxes (Fig. 66), the surface P is covered with lamp-black, T is made of bright metal. A box containing hot water is placed between and equidistant from them. If the surface R is bright but S is lamp-blacked it is found that there is no movement of the index when the box is placed in position. If Q is the heat emitted from S and A the absorptive power of T, then the heat absorbed by the right-hand bulb is AQ. Also if E is the emissive power of R the heat it emits is Q′ where, from the definition of emissive power (p. 125), $Q'/Q = E$ or $Q' = EQ$. The whole of this is absorbed by P, hence, as the index does not move, $EQ = AQ$ or $E = A$.

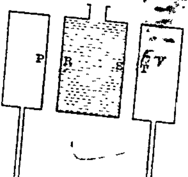


Fig. 66.—Ritchie's Apparatus.

Prevost's Theory of Exchanges. Stefan's Law.—The experiments described in the last paragraph show that a thermometer has a steady temperature when its losses arising from emission are just balanced by the radiation it absorbs from surrounding bodies. According to Prevost's theory of exchanges this is true for every case of temperature equilibrium. When a hot body is brought near a thermometer which is emitting radiation, and the temperature of the thermometer rises because it receives more than it loses. Similarly when a block of ice is brought near, the temperature of the thermometer falls because it does not gain as much radiation from the ice as it sends to it. Stefan has shown that the amount of radiation emitted by a body is proportional to the fourth power of its absolute temperature for a perfectly black body this law has been proved to hold over a very wide range. Taking the case of a black body at an absolute temperature T_1 placed in an enclosure whose walls, also black, are at absolute temperature T_2, the heat the body loses by radiation in unit time is cT_1^4, where c is a constant depending on the nature

ture. Thus in the same time it gains radiation from the sur
σT_2^4; the rate of cooling is therefore $\sigma(T_1^4 - T_2^4)$. If T_1
greater than T_2 this is practically equal to σT_1^4. Suppose
other hand, T_1 and T_2 are nearly equal so that $T_1 = T_2 + t$
very small compared with either. The rate of cooling

$$\sigma\{(T_2 + t)^4 - T_2^4\}$$

But $(T_2+t)^4 = T_2^4\left(1 + \frac{t}{T_2}\right)^4$

$$= T_2^4\left(1 + 4\cdot\frac{t}{T_2} + \text{terms containing higher powers of } \frac{t}{T_2}\right)$$

by the binomial theorem.

As t/T_2 is small these higher powers may be neglected, and th
rate of cooling is

$$\sigma T_2^4\left(1 + 4\frac{t}{T_2}\right) - \sigma T_2^4 = 4\sigma t T_2^3$$

This shows that the rate of cooling is proportional to the tempe
ture excess t in accordance with Newton's law. The calcula
shows clearly that Newton's law of cooling can hold only when
temperature of the hot body is slightly above that of its surround
To calculate the rate of cooling in other cases Stefan's law mu
used.

EXAMPLES ON CHAPTER XII

1. In what respects does radiant heat differ from light? Why a
salt lenses employed for experiments with radiant heat coming from a
at a low temperature, while glass lenses suffice when the sun or an electri
is employed as a source of heat? (L. '80.)

2. How are the radiating and absorbing powers of a surface conn
Describe experiments to verify the connection. (L. '88.)

3. A piece of ice is placed in front of a thermopile and the needl
galvanometer is seen to move. Describe as fully as you can all that
on. (L. '90.)

4. How do you account for the fact that on a frosty night it is ofte
the bottom of a valley than on the neighbouring hill sides? (L. '
5. How would you show that a large amount of the energy radia
consists of non-luminous heat rays, and how would you me
... place? (L. '09.)

CHAPTER XIII

RECTILINEAR PROPAGATION OF LIGHT

word "light" is used in two senses; we speak of the se
ght, and the same term is used to denote the physical ca
sensation. It is in the second sense that the word is used
ing pages.

metrical and Physical Optics.—Light may be studied from
of view; in the first method certain simple laws are first es
y experiment, and from them by mathematical and phys
g we proceed to deduce other, probably more complicat
From this standpoint we are not concerned with t
ysical nature of light, nor with the reason why the fundament
s are obeyed: this branch of the subject is called Geometric
ics. In the second method an attempt is made to go further an
rm some hypothesis as to the nature of light; from this certair
equences are deduced which can again be subjected to the test
periment. This is the province of Physical Optics. The two
ds cannot be kept separate without falling into error. In the
ing pages light is studied by the first, or geometrical, method,
is advantageous to assume one of the chief results of the
l optical theory, viz. that light consists of extremely short
Any substance through which light travels is called a
; this term also includes the non-material ether which is
to fill all space (p. 116). Bodies which emit light are called
ous bodies. They are known to contain certain particles
rate rapidly to and fro, thus setting up disturbances in
ing media which, for want of a better name, are called waves.
distance which a wave travels while the particle makes one
plete vibration is called the wave-length. These wave-lengths
xtremely small; if they lie between 4×10^{-5} and 8×10^{-5} cms
waves produce the sensation of sight when they ...

[illegible] as waves of [illegible] and [illegible] waves [illegible] are also given [illegible] the laws governing [illegible] propagation are the same as for light waves [illegible] of light is based [illegible]

The three laws on which the study of [illegible] of light is based are

(1) The rectilinear propagation of light, or the fact that light travels in straight lines.

(2) The laws of reflexion.

(3) The laws of refraction.

Rectilinear Propagation of Light.—Some of our common notice[illegible] are based on the assumption that light travels in straight line[illegible] Thus in [illegible] we point it in the direction in which the li[illegible] reaches us from the object mark, and we assume that the moon [illegible] actually situated in the direction in which it is seen.

Experiment. Place three pieces of cardboard in vertical positions beh[illegible] each other and [illegible] apart. Make a small hole in each and arrange th[illegible] in the same straight line by threading the cards on a knitting needle. [illegible] a lamp is placed behind the first hole light travels through each of the [illegible] and may be received on a screen placed behind the last. If one scre[illegible] slightly displaced sideways light no longer gets through.

In a darkened room which has a small hole in the shutter [illegible] and particles of dust floating about; [illegible] path of the light is shown by particles of dust floating about; [illegible] seen to be a straight line.

The pin-hole camera provides a simple illustration of the [illegible] A candle is placed a short distance behind a screen of [illegible]oard in which a small pin-hole has been made; light tra[illegible] traight lines from the different points of the flame, passes th[illegible] the hole, and falling on a screen behind produces a series of illum[illegible] patches. Fig. 67 shows how this results in an inverted picture [illegible] flame being formed on the screen. The size of this picture [illegible] dently depend on the relative distances of screen and cand[illegible] the hole. If a second pin-hole is made, not far from the first, [illegible] candle flame will be seen on the screen; if the two over[illegible] resultant picture will be blurred. Hence we see that a la[illegible] [illegible]ich we may regard as a number of small holes near toge[illegible] [illegible]t produce a clear picture. If three pin-holes are ma[illegible] [illegible] to form a small triangle, the three pictures [illegible] [illegible] that there is litt[illegible]

learness, *i.e.* the shape of the small hole does not affect the form of he picture; but if the screen is close to the hole the picture produced y each part is very small, and, as there is very little overlapping, an lluminated spot is seen of the same shape as the hole itself.

A homogeneous medium is one whose properties do not vary rom point to point. It will be seen later that when light travels rom one medium to another its path is usually bent at the surface of separation; bearing this in mind the first law of Geometrical

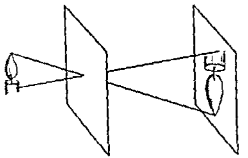


FIG. 67.—The Pin-hole Camera.

Optics can be stated in the following terms: **Light travels in straight lines in a homogeneous medium.**

Definition of Terms.—The straight lines along which light travels are called rays. A collection of rays forms a beam or pencil of light. If the rays converge to or diverge from a point the beam is said to be convergent or divergent respectively; when the rays are parallel we have a parallel beam. Rays diverge in all directions from any point of a luminous body, but when it is very distant the rays with which we deal are inclined at such a small angle that they may be regarded as parallel. Thus the rays coming from a star to the eye form a parallel beam.

Shadows.—The formation of shadows is a direct consequence of the fact that light travels in straight lines.

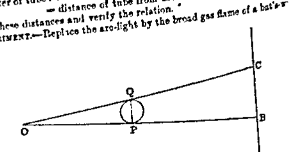
EXPERIMENT.—Take a small source of light, such as the arc light, fix it some distance away from a vertical piece of brass tube several inches in diameter, and notice that a well-defined shadow is thrown on a screen held a few feet away.

Fig. 68, A, shows how this is produced. The opaque tube prevents any light from the arc reaching the part BC of the screen, an eye placed between B and C would not see the arc. Evidently if light travels in straight lines we sha[ll] have from the triangles OBC, OPQ.

Diameter of tube : width of shadow
= distance of tube from arc : distance of screen from a[rc].

Measure these distances and verify the relation.

EXPERIMENT.—Replace the arc-light by the broad gas flame of a bat's-w[ing]



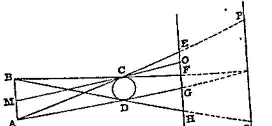


FIG. 68, A and B.—Illustration of how Shadows are formed.

burner. The shadow is now clear at the centre but becomes indistinct [at] the edges.

Fig. 68, B, shows how the shadow is formed when such an ex[tended] source is used, AB represents the flame, CD the brass tube a[nd] the screen. Any straight line drawn from the flame to a p[oint of] the screen between F, G, must pass through the tube. He[nce the] part FG receives no light, it is the region of complete sha[dow or] umbra. Outside this there are regions EF, GH, which rece[ive light from] some parts of the flame but not from the whole; they [are] [reg]ions of half-shadow or penumbra. Thus the part OF rec[eives]

ght from any part of the flame nearer to A than M, the rays startin rom AM in the direction of OF are stopped by the tube. If a observer looks towards the flame through a hole at O he will see onl he part BM, if the hole is between F and G no part of the sourc will be seen. Beyond E and H the screen is fully illuminated. Th difference between this and the preceding case is seen to be due t the extended source of light that is used. The relative and actu sizes of umbra and penumbra depend on the relative positions source, tube and screen. If the screen is placed near the tube th penumbra is small, while if it is placed at PQ there is scarcely an umbra.

Eclipses are results of the formation of shadows by the moon the earth. It happens at certain times that the moon moves into position between the sun and some portion of the earth's surfac the sunlight is intercepted and the sun is said to be eclipsed. points on the earth which are in the umbra the eclipse is total, whe only the penumbra occurs the eclipse is partial. Fig 68, B, illustrat what may happen if AB is taken to represent the sun, CD the moo and EH the earth. Lunar eclipses are caused by the earth getti into a position between the sun and moon. Fig. 68 B illustrat this case if CD now represents the earth, and part of the screen F the moon. The moon is not self-luminous, the light we receive fro it is reflected sunlight, hence if it is in the shadow cast by the ear no light can be sent back and it is eclipsed. If it is in the umb FG in the figure, the eclipse is total, if in the penumbra a part eclipse takes place.

EXAMPLES ON CHAPTER XIII

1. On a clear, sunny day a flag-staff casts a shadow on the ground and is found that the portion due to the lower end is the best defined; explain t How would white clouds affect the shadow?

2. Why are well-defined circular shadows sometimes seen on the gro beneath an arc lamp?

3. A strip of wood 1 cm. in width is held in a vertical position betwee gas flame and a screen, its distance from the former is 60 cms. and from latter 30 cms. If the flame is 2 cms. wide find the diameter of the umbra the width of the penumbra on one side of the shadow.

CHAPTER XIV

REFLECTION OF LIGHT FROM PLANE SURFA[CES]

Diffusion and Reflection. When light falls on the surfa[ce of separa-]tion of two media which differ in their optical propert[ies, the rays] are divided into two portions. One part is returned i[nto the first] medium and is said to be reflected; another part enter[s the second] medium and is either absorbed if the medium is opaque, [or trans-]mitted if it is transparent. We will confine our attentio[n at present] to the reflected rays. The directions in which these ra[ys travel are] governed by definite laws, the laws of reflection. When [the surface] of the second medium is not highly polished each of the s[mall irregu-]larities on it will reflect light, and, as these irregular surf[aces are] inclined at all angles, rays will be reflected in all direc[tions. The] light is then said to be diffused or scattered. It is owin[g to diffused] light that the surfaces of bodies are visible; thus it is [difficult to] see the surface of a brightly polished mirror because of [the absence] of diffused light; it is much easier to see if we breathe upon [it. By taking] advantage of diffusion we can easily make apparent the p[...] in a transparent medium.

EXPERIMENT.—Fill a flask with distilled water which has b[...] and focus on it a beam of light from a lantern. The path of the [...] with difficulty, but the addition of a few drops of milk renders [...] to follow because the milk particles diffuse light in all directions. [...] is one of the most delicate tests for the presence of suspended part[icles ...] may appear at first sight to be a homogeneous liquid. Note also th[at the path] of the beam from the lantern is made visible in the air by the d[ust particles] floating about in it; a cloud of smoke makes it still clearer o[wing to the] increased number of diffusing particles.

[illegible]

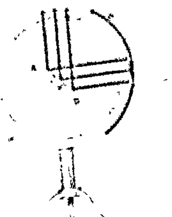


Fig. 7[illegible] [illegible]

[illegible] straight line with P, Q. [illegible] the normal ON with [illegible] A number of rays [illegible] be traced in this manner an[illegible] corresponding angles found. [illegible] of the paper the planes of [illegible] points of the four pins are in the plane of the paper [illegible] of a plane mirror [illegible] ed [illegible]

Images.—When a pin is held in front of a plane mirror [illegible] reflecting surface [illegible] picture of it which appears to be behind the reflecting surface [illegible] picture is called the image of the pin. If rays of light starting [illegible] one point afterwards appear to pass through another poi[illegible] second point is called the image of the first; if the rays ac[illegible] pass through the second point it is a real image, if they only [illegible] to pass through it the image is called virtual. In the examp[illegible] given the image of the pin is behind the mirror, as the rays [illegible] actually come from this place, but only appear to do so on [illegible] of reflexion, the image is virtual. The images formed by a [illegible] camera are real. We shall meet with other instances of eac[illegible]

Experiment.—*To find the position of a virtual image formed by* [illegible] mirror. Use the apparatus of the last experiment. Fix a pin at P a [illegible] in front of the mirror (Fig. 71); it sends out rays in all directions [illegible] required to find the point from which these rays appear to diverge after [illegible] ... R, just in front of the mirror and place a third p[illegible]

uch a position that S, R, and the image of P all appear to be in the same traight line. RS is one reflected ray. Move the pin R to the right or left and find other reflected rays in a similar manner. Rule in the position of the mirror, then remove it and produce the reflected rays backwards; they will meet approximately at a point which is the position of the image. Measurements will show that the image P′ lies on that normal to the mirror which passes through P, and is as far behind the mirror as the object is in front. Since the light is reflected chiefly from the back surface the measurements must be made from this edge.

The result obtained from this experiment should be remembered.

Method of Parallax.—This is a method of finding the position of an image when it is impossible or inconvenient to trace the paths of

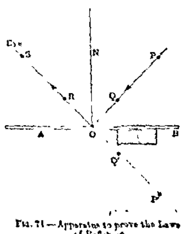


FIG. 71.—Apparatus to prove the Laws of Reflexion.

FIG. 72.—To illustrate Parallax.

the rays. It consists in placing a pointer so that it does not appear to shift relatively to the image when the observer changes his position, image and pointer then coincide.

EXPERIMENT.—Stick a pin vertically in a drawing board. [illegible] behind it fix a larger pin. Shut one eye and [illegible] that the pins appear in the [illegible] sight; the [illegible] relatively [illegible] Fig. 72 [illegible]

appears to come from P′. Join P′ to the edges of the eye-pupil. The part P′S of the pencil has no actual existence. Join the points where the rays cut the mirror to P, the pencil PSE shows the path of the rays. In the same manner we can construct the path of the rays by which the image of an extended object is seen; the rays from each point must be found separately. If the object P is placed between two parallel mirrors A, B (Fig. 75), a succession of images is seen. Thus in mirror A an image is formed at P_1, where $PA = P_1A$; the reflected rays now appear to come from this image and when they fall on the second mirror they will form a further image at P_2 as if they actually came from P_1; hence $P_1B = P_2B$. Similarly P_2 gives rise to an image P_3, where $P_2A = P_3A$, and so on. Another series is formed starting from the mirror B; the first is at Q_1 where

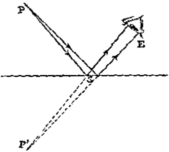


FIG. 74.—Showing the Path of the Rays to the Eye.

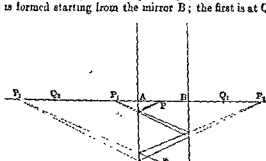


FIG. 75.—Formation of Images by Parallel Mirrors.

$B = Q_1B$, this is imaged in A at Q_2, hence $Q_1A = Q_2A$, etc. Let us draw the rays by which a given image, say P_3, is seen. Draw from it a divergent pencil entering the eye at E; before their final

appears to come from P′. Join P′ to the edges of the eye-pupil. The part P′S of the pencil has no actual existence. Join the points where the rays cut the mirror to P, the pencil PSE shows the path of the rays. In the same manner we can construct the path of the rays by which the image of an extended object is seen; the rays from each point must be found separately. If the object P is placed between two parallel mirrors A, B (Fig. 75), a succession of images is seen. Thus in mirror A an image is formed at P_1, where $PA = P_1A$; the reflected rays now appear to come from this image and when they fall on the second mirror they will form a further image at P_2 as if they actually came from P_1; hence $P_1B = P_2B$. Similarly P_2 gives rise to an image P_3, where $P_2A = P_3A$, and so on. Another series is formed starting from the mirror B; the first is at Q_1 where

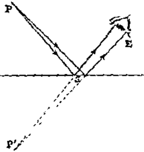


Fig. 74.—Showing the Path of the Rays to the Eye.

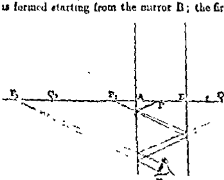


Fig. 75.—Formation of Images by Parallel Mirrors.

$PB = Q_1B$, this is imaged in A at Q_2, hence $Q_1A = Q_2A$, etc. Let us trace the rays by which a given image, say P_3, is seen. Draw from it a divergent pencil entering the eye at E; before their final

circular hole; the wires are then at the centre of curvature, for $v = u$ in t[he] ordinary formula, hence $u = r$.

EXPERIMENT.—Use the same apparatus but move the mirror further fr[om] the cross-wires; a real image is formed as in Fig. 83. This may be recei[ved] on a small screen placed between the mirror and source. The distance of [the] cross-wires and image from the mirror are u and v of the formula $\frac{1}{v} + \frac{1}{u} =$ whence f can be found; different positions of the mirror should be used. performing the calculations much time is saved by a set of tables giving reciprocals of numbers. Thus in an experiment $u = 85{\cdot}6$ cms., $v = 42{\cdot}4$ c[ms.] from tables,

$$\begin{aligned} \text{reciprocal of } 85{\cdot}6 &= 0{\cdot}01168 \\ \text{,,} \quad \text{,,} \quad 42{\cdot}4 &= 0{\cdot}02359 \\ \text{sum} &= 0{\cdot}03526 = \frac{1}{f} \end{aligned}$$

$\therefore f =$ reciprocal of $0{\cdot}03526 = 28{\cdot}5$ cms. from the tables.

If the formulæ are correct we should get the same value of th[e] focal length by all methods, within the limits of experimental erro[r].

To verify the formulæ for magnification the cross-wires are r[e]placed by a rectangular slit about 2 cms. wide and the sizes of th[e] image and of the slit are measured. If in addition to measuring th[e] magnification the distance of image or object from the mirror [is] found we can calculate f from the formulæ

$$\text{magnification} = \frac{v - f}{f} = \frac{f}{u - f}$$

With convex mirrors there is the difficulty that the image is vir[tual and] therefore cannot be received on a screen. The following meth[od may] be used in this case (see also p. 199).

... the cross-wires on the bench, ... reflecting face tow...

is equal to the distance of the second needle from B and this can be measured directly, also $AQ = BQ - BA = v$ and $PA = u$.

Reflexion of Waves other than Light Waves.—For experiments on these waves a thermopile must be used as detector, since they do not cause the sensation of sight. The position of the image formed by a concave mirror has been deduced from the laws of reflexion, if therefore it is found that other radiations (so-called heat waves, p. 116) are brought to a focus at the same point, these radiations must obey the same laws. Concave metal mirrors may be used in place of glass.

EXPERIMENT.—Find the focal length of such a mirror for rays of light, then use as source of radiation a Bunsen burner with a rose top placed some distance

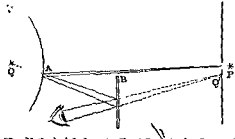


FIG. 87.—Method of finding the Focal Length of a Convex Mirror.

in front of the mirror. Let the reflected rays fall on a thermopile which is connected to a sensitive galvanometer, move this up to the mirror until the maximum deflexion of the needle is produced. The image of the burner is now on the blackened junctions of the thermopile; if the distances of image and object from the mirror are measured the focal length can be calculated from the usual formula. It will be found to be practically equal to that found by optical methods. As the conical reflector on the thermopile collects rays which would not otherwise fall on the junctions it is best removed for this experiment.

EXPERIMENT.—Arrange two concave mirrors to face each other about 10 ft. apart; fix at the focus of one a thermopile, at the focus of the other a lighted candle or a Bunsen burner. The rays from the source are made parallel by reflexion at one mirror and are focussed by the other on the thermopile; a considerable deflexion is produced, but this is greatly reduced if either source or thermopile is moved to one side.

EXPERIMENT.—Arrange two long brass tubes 2- in diameter as shown in Fig. 88, place a rose burner at the end of on e corre-sponding end of the other. Pro from direct radiation from the fl cir-cumstances. Place a re en they

These experiments show that the ... according to the same laws as light.

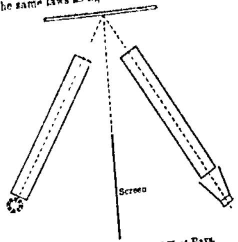


FIG. 88.—Showing Reflexion of Heat Rays.

Measurement of an Angular Deflexion.—Instead of the method of measuring an angular deflexion a concave mirr

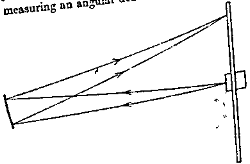


FIG. 89.—Lamp and Scale Method of measuring an Angular Deflexion

1. The mirror (Fig. 89) is attached to the body whose defle ... telescope is replaced by a narrow vertic

ninated from behind by a lamp. The slit is fixed at a distance n the mirror equal to the radius of curvature, and immediately ve or below it a divided scale is placed. In these circumstances image of the slit is thrown on the scale, and the movements of spot of light are used as in the previous method to measure the lexion of the mirror. It must be remembered that the angle ough which the reflected ray is turned is twice the angular dis-cement of the mirror.

EXAMPLES ON CHAPTER XV

1 A candle flame is placed 25 cms. away from a concave mirror whose lius of curvature is 80 cms.; find the position and nature of the image.

Here $u = 25$, $f = 40$,

$$\therefore \frac{1}{v} + \frac{1}{25} = \frac{1}{40}$$

ience, from a table of reciprocals,

$$\frac{1}{v} = 0.025 - 0.04 = -0.015$$

Hence $\qquad v = -66.7$ cms.

The magnification $\qquad -\frac{v}{u} = \frac{66.7}{25}$

he image is therefore 66.7 cms. behind the mirror and is virtual, enlarged, nd erect (magnification positive).

2. Where must the candle be placed in order that a real image, five times ge as the object, may be formed?

the image is real v and u have the same sign and $v/u = 5$, or $v = 5u$.

ince

$$\frac{1}{5u} + \frac{1}{u} = \frac{1}{40}$$

$$u = 48 \text{ cms.}$$

In what positions must the candle be placed to give rise to an image times as large as the object?

An object 2 cms. in length is placed 35 cms. in front of a mirror and the image is found to be 4 mms. high; find the focal length of the mirror. t is the focal length if the image is virtual?

When a gas flame is placed 32 cms. in front of ... It is found that image is 12 cms. behind the mirror. Find the ... ature.

Show that the image formed ...

hen the angle of incidence is nearly 90° a refracted ray is possible. t is otherwise when the first medium is denser than the second. Thus in Fig. 90, if the light is travelling in the direction QNP and he angle of incidence O'NQ is gradually increased, a stage is arrived t where the angle of refraction is 90°, the refracted ray then travels parallel with the surface of the glass. If the angle of incidence is ncreased still more there is no refracted ray, all the light is reflected back again into the first medium; this is called **total internal reflexion.** The angle of incidence at which total reflexion begins is called the **critical angle.** The table on p. 165 shows that the critical angle for the glass used is 41.5°. If μ_{ga} is the refractive index from glass to air and θ is the critical angle

$$\mu_{ga} = \frac{\sin\theta}{\sin\frac{\pi}{2}} = \sin\theta$$

hence the refractive index of the glass

$$\mu_{ag} = \frac{1}{\mu_{ga}} = 1/\sin\theta = \operatorname{cosec}\theta$$

In Fig. 95 the ray OQ is incident at the critical angle, the ray OS is totally reflected. It follows from this that if we stand on the side of a swimming bath we shall not be able to see the more distant points at the bottom, the rays coming from such points in the direction of the eye are totally reflected at the surface of the water. A crack in a window-pane looks brightly reflecting for a similar cause; rays travelling in the glass strike the air film at an angle greater than the critical angle and are totally reflected.

EXPERIMENT.—Hold an empty test tube in an inclined position in a beaker water and view it from above. The sides of the tube look like a brightly lished mirror owing to the light which comes through the sides of the beaker ing totally reflected.

Whenever light is reflected from a glass mirror a certain amount at is lost since part is refracted; this loss can be avoided by making se of total reflexion. Fig. 96 shows a total reflexion prism in a orm frequently used to turn the path of a beam of light through 90°. t is a right-angled isosceles prism of glass having a refractive index bout 1.51; for such a glass, as the table on p. 165 shows, the ritical angle is 41.5°. Rays fall normally on one of the short faces

between the second and third readings the length O'N, μ, can therefore be found. The same method can be applied to find the refractive index of a liquid. A piece of marked paper is stuck to the inner side of the bottom of a beaker and the microscope focussed on it as before. Liquid is poured in and the microscope focussed in succession on the mark and on the upper liquid surface. The calculation is made as in the last case.

It is only when we limit ourselves to the rays emerging near N that a definite image is formed. If we find the position of a numbe

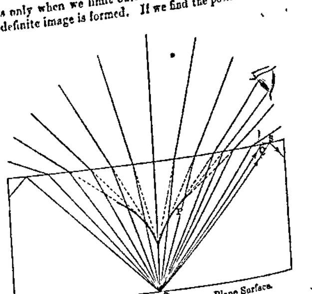


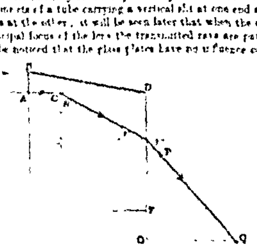
FIG. 95.—Caustic by Refraction at a Plane Surface.

of refracted rays, by the pin method given above, results are obtained similar to those shown in Fig. 95. The extreme rays do not diverge even approximately from a point, and the apparent thickness of the block varies with the position of the observer. Thus if the eye is placed as shown in the figure the pin appears to be at P. The curve joining the points of intersection of adjacent refracted rays is called the caustic by refraction; it is shown by the thickened line in the figure.

Total Reflexion.—When light passes from a rare into a dense medium the refracted ray is bent towards the normal, so that

he critical angle θ, hence this can be found from the graduated circle by noting the positions of the pointer when the light is suddenly cut off then $\mu = \operatorname{cosec}\theta$. In order that the light shall disappear suddenly it is necessary that all the rays shall strike the film at the same angle, to ensure this they are made parallel by means of a collimator.

This consists of a tube carrying a vertical slit at one end and a convex lens at the other; it will be seen later that when the slit is at the principal focus of the lens the transmitted rays are parallel. It should be noticed that the glass plates have no influence on the



and travel on without refraction; they meet the su[rface] ... of an
[angle] of 45°, which is greater than the critical angle, and are then totally reflected without loss.

Application of the Critical Angle to the measurement of Refractive Index. The last paragraph shows that the refractive index of the denser medium referred to air is given by $\mu = \operatorname{cosec} \theta$, where θ is the critical angle for light travelling from the medium into air. This equation provides a simple and at the same time most accurate method of measuring refractive indices. It is the method which is most commonly used for liquids. Fig. 97 shows a section of a simple form of apparatus which may be used. D is a rectangular glass vessel containing the experimental liquid; in this is immersed a thin film of air AB enclosed between two glass plates. This part of the apparatus is made by separating two glass plates at their edges by a narrow strip of tin foil and smearing their whole perimeter with

FIG. 96.—Total Reflection in Prism.

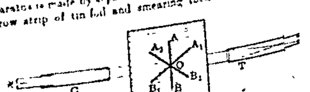
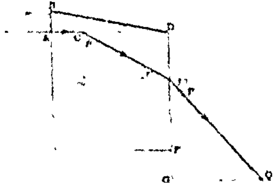


FIG. 97.—Measurement of Refractive Index by Total Reflexion.

shellac varnish so as to make an air-tight seal. The plates are supported by a vertical rod which carries a horizontal pointer moving over a graduated circle. Light coming from a salted Bunsen flame passes into the liquid and through the film into a telescope. Suppose the air-film to be initially in the position AB perpendicular to the path of the light; the normal to AB is along the rays and the angle of incidence on the film is zero. If the plates are turned, when this angle of incidence is equal to $\angle AOA_1$; when this is A_1B_1 the angle of incidence is equal to $\angle AOA_1$; when this is the critical angle for water-air no light enters the telescope. The plates are next turned through their first position into that shown at ... reflexion again occurs. Evidently the $\angle A_1OA_2$ is

the critical angle θ, hence this can be found from the graduated circle by noting the positions of the pointer when the light is suddenly cut off, then $\mu = \operatorname{cosec} \theta$. In order that the light shall disappear suddenly it is necessary that all the rays shall strike the film at the same angle; to ensure this they are made parallel by means of a collimator.

This consists of a tube carrying a vertical slit at one end and a convex lens at the other; it will be seen later that when the slit is at the principal focus of the lens the transmitted rays are parallel. It should be noted that the glass plates have no influence on the



surface at an angle just less than 90° are refracted into the glass at the critical angle θ. For these emergence and travel in the direction PQ. All other rays enter the glass at an angle of refraction smaller than θ. Just as the eye of an observer focusses into a point all the parallel rays coming from a star, so will an eye placed at Q focus into a straight line all the rays which emerge parallel to PQ. The face AD of the cube will therefore be divided into bright and black halves, and the rays coming from the line of division enter the glass at an angle θ. Let N and μ be the refractive indices of glass and water respectively, μ_{wg} be the refractive index going from water to glass.

$$\mu_{wg} = \frac{N}{\mu} \quad \text{(p. 197)}$$

Then

$$\mu_{wg} = \frac{\sin\frac{\pi}{2}}{\sin\theta} = \frac{1}{\sin\theta} \quad \text{(from the figure)}$$

$$\therefore \sin\theta = \mu/N$$

Also

$$N = \sin i/\sin r \text{ (see figure)}$$

But and as the angle at D is a right angle

$$\sin r = \cos\theta$$

$$\therefore N = \frac{\sin i}{\cos\theta}$$

or

$$\cos\theta = \frac{\sin i}{N}$$

Squaring and adding the expressions for $\sin\theta$ and $\cos\theta$ we get

$$\frac{\mu^2}{N^2} + \frac{\sin^2 i}{N^2} = \sin^2\theta + \cos^2\theta = 1$$

or

$$\mu^2 + \sin^2 i = N^2$$

Hence if the angle i is measured we can calculate either μ provided the other is known. Usually we shall know N, and the apparatus can then be used for different liquids. To measure are placed at P and Q, about 30 cms. apart, so that they app line with the dark edge of the field, a ruler is placed along D this straight line produced. From Q the line QG is drawn p ...dicular to DF, then cos QPG = sin i can be found by measur

l the front and back surfaces of the mirror are parallel, the images not seen, for in that case the incident and therefore the emergent s are parallel, hence they appear to come from a single distant ige. If in any instance a distant candle is used and the series of iges is still visible it shows that the faces of the mirror are not rallel; this, in fact, provides a simple means of testing the rallelism of the front and back faces.

EXAMPLES ON CHAPTER XVI

1. An object is viewed through a thick plate of glass so that the rays meet e plate at nearly normal incidence. Prove that its apparent displacement wards the observer is independent of its small distance from the glass.

2. A ray of light passes obliquely through a plate of glass with parallel les. Show that the distance between the emergent ray and the incident y produced is $\frac{e \sin(i - r)}{\cos r}$, where e is the thickness of the plate, and i and r e the angles of incidence and refraction respectively.

3. Explain why a thick plate of glass produces no appreciable displacement the apparent position of a distant object viewed through the plate. The ys are supposed to meet the plate normally. (L. '88.)

4. A substance has a refractive index $\sqrt{3}$. Draw as nearly as you can to ale the path of a ray incident on a parallel plate of the substance 1 in. hick, the angle of incidence being 60°. What is the distance between the cident ray produced and the emergent ray? (L. '95.)

5. A pencil of light from a point is incident on a plate of a refracting subtance. Show that, if the pencil is nearly normal, then within the plate it oceeds as if it came from an image μ times as far from the surface as the minous point. Draw a figure for the case in which $\mu = 2$. (L. '96.)

6. Draw to scale a diagram showing the directions in water in which a ray light, incident at 45° on the surface of the water, will travel, assuming that refractive index of water is $\frac{4}{3}$. (L. '07.)

7. Show that if a horizontal concave mirror is filled with liquid its apparent lius of curvature is diminished in the ratio of the refractive index of the uid. (L. '07.)

8. Prove that to an eye under the surface of water all objects that can seen above the surface appear in a cone whose semi-vertical angle is the itical angle.

9. A cubical block of glass is placed on a black glass plate with a film of uid between them and the whole is placed before a window with one face of e cube vertical. On looking through the opposite vertical face from a posit...

10. If the eye-piece is drawn out from the ... [illegible] ... of the lens before ... [illegible] ... that $\mu^2 = ...$ [illegible] ... glass ... [illegible] ... water and ... glass ... [illegible] ... and ... are the refractive indices of water and ... [illegible] ... in the lens ... [illegible]

12. The refractive index of water and ... [illegible] ... glass ... [illegible] ... on the bench, a piece of ... [illegible]

13. A ... microscope is focussed on a mark on the bench ... Find how much the microscope must be ... glass 3 cm. thick is then interposed. ... [illegible] ... moved for the mark to be in focus ... (μ of glass $= 1.5$)

14. A beaker containing liquid is placed on the table underneath a microscope which can be moved along a vertical scale. The microscope is focussed through the liquid on to a mark on the table when the reading on the scale is a. It is next focussed on the upper surface of the liquid and the reading is b. More liquid is added and the observations are repeated, the corresponding readings being c and d. Show that the refractive index of the liquid is

$$\frac{d-b}{a+d-b-c}$$

CHAPTER XVII

APPLICATIONS OF THE LAWS OF REFRACTION

ssage of Light through a Prism.—A portion of a medium between o plane faces inclined to each other at an angle is called, for optical rposes, a prism. The line of intersection of the faces is called the fracting edge, and a section perpendicular to this line is a principal ction. The angle between the faces is called the angle of the ism. In what follows we shall deal only with rays in a principal ction and we shall further suppose the light is such as is obtained om a salted Bunsen flame. The general features attending the ssage of light through a prism are best studied on the Hartl disc.

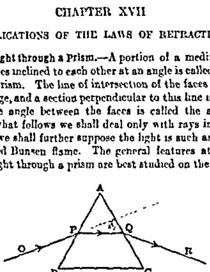


FIG. 102.—Path of Light through a Prism.

EXPERIMENT.—Fix a prism APC (Fig. 102) on the disc and arrange the slit so hat a ray OP falls on one face; the path through the prism is shown by OPQR, nd the ray is bent away from the refracting edge. The angle between the initial and final directions of the ray, δ in the figure, is called the deviation produced by the prism. If the prism is rotated round the point A the deviation changes; turn it continually in that direction which causes the deviation to diminish, it is found that the emergent ray QR gradually approaches a direction parallel to OP, but before reaching this position it stops and finally moves back again. Hence for a particular angle of incidence the deviation is a minimum, when this is reached it can easily be shown by measurement that the $\angle OPB = \angle RQC$, i.e. in the minimum deviation position the light passes symmetrically through the prism.

EXPERIMENT.—To plot a curve showing how the deviation varies with the angle of incidence on the first face. Pin a sheet of paper on a drawing board and draw lines OY, YY' at right angles to each other (Fig. 103). Draw a number of lines OP₁, OP₂, etc., making angles of 10°, 20°, etc., with OX. Place the prism with one face along OX, the refracting edge being at A. Let YO represent a ray incident normally on the face AB. This ray travels straight along to [illegible] and, unless total reflection occurs, emerges in the direction NM. The angle between NM and OY is the deviation produced. Stick two pins some distance apart on OY, and, looking through the prism in the direction MN, fix two other pins at M and N to be in the same straight line as the images of the first two. The line joining the second pair of pins is the emergent ray. Remove

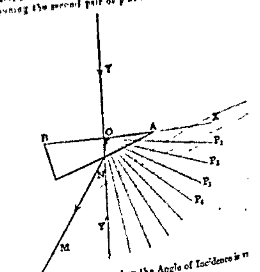


FIG. 103.—Change in the Deviation when the Angle of Incidence is [illegible]

the prism and measure the deviation with a protractor.[1] Next fix [illegible] with its face AB along OP₁, the angle of incidence of the ray along [illegible] and the emergent ray and deviation can be found as before. Repeat [illegible] face AB along OP₂, OP₃, etc.; plot a curve showing the deviation for [illegible] angles of incidence. Read from your curve what is the angle of [illegible] minimum deviation, place the prism in the corresponding position [illegible] the emergent ray. By joining the points at which the incident and [illegible] the prism get the path of the ray in the glass; show [illegible] [illegible]ined to the prism faces. Rule in the outline of the [illegible] [illegible]gle for future use.

[1] [illegible]ted to Mr. F. J. Harlow for this method of carry[illegible]

EXPERIMENT.—Trace a ray through a prism as in the last experiment, draw the normal to one face at the point where the ray intersects it and measure the angles of incidence and refraction; hence calculate the refractive index of the glass.

Image produced by a Prism.—Let P (Fig. 104) represent a source of light and let the ray PQ pass through a prism with minimum deviation. Two near rays PR, PS, are incident at slightly different angles, but an inspection of the curve obtained above shows that near the minimum the deviation varies very slowly with the angle of incidence, hence the deviation of these rays is practically equal to that of PQ. It follows that the inclination of the rays to each other is unaltered by their passage through the prism, hence if the

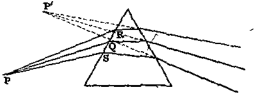


FIG. 104.—Formation of an Image by a Prism.

emergent rays are produced backwards they will meet at a point P′ whose distance from the prism is equal to that of P. (This is not quite true unless the thickness of the glass is neglected, P should be a considerable distance away.) P′ is the virtual image of P. When the prism is in any other position the corresponding rays are unequally deviated, they no longer diverge from a point after refraction and no true image is formed. Whenever it is necessary to produce a well-defined image the prism must be placed in the minimum deviation position.

EXPERIMENT.—Use a vertical pin as object and trace rays through a prism. Show that the emerging rays do not diverge from a point except in the minimum deviation position. Fix another pin by the parallax method to coincide with the image in the latter case; for this purpose the second pin must be long enough to be seen over the top of the prism. Show that image and object are equidistant from the first face.[1]

Measurement of Refractive Index by means of a Prism.—Let PQRS (Fig. 105) be the path of a ray through a prism and δ the deviation produced. Draw the normals QN, RN to the prism faces and let

[1] Barton and Black, "Practical Physics," p. 87.

the angles of incidence and refraction be as shown in the figure.
Then from $\triangle$ OQR

$$\begin{aligned}\delta &= \angle OQR + \angle ORQ \\ &= (\angle OQN - \angle RQN) + (\angle ORN - \angle QRN) \\ &= (i_1 - r_1) + (i_2 - r_2) \\ &= i_1 + i_2 - (r_1 + r_2)\end{aligned}$$

But the interior angles of the quadrilateral AQNR together equal four right angles, and as the angles at Q and R are right angles

$$\angle A + \angle N = 2 \text{ rt. } \angle s$$

Also

$$r_1 + r_2 + N = 2 \text{ rt. } \angle s$$

$$\therefore A = r_1 + r_2$$

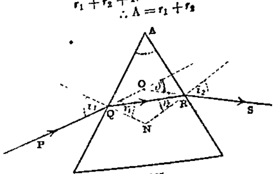


FIG. 105.

In the minimum deviation position $i_1 = i_2$ and $r_1 = r_2$

$$\therefore A = 2r_1$$

and

$$\delta = 2i_1 - 2r_1 = 2i_1 - A$$

$$\therefore i_1 = \frac{\delta + A}{2}$$

and

$$r_1 = \frac{A}{2}$$

Now the refractive index of the prism material

$$\mu = \sin i_1/\sin r_1$$

$$\therefore \mu = \frac{\sin \dfrac{\delta + A}{2}}{\sin \dfrac{A}{2}}$$

This equation shows that μ can be calculated when the angle of the prism and the minimum deviation have been measured. An accurate method of making these measurements will be given later.

EXAMPLE.—The minimum deviation and angle of a prism have been measured in a previous experiment (p. 180), calculate the refractive index from these results.

When the prism angle is very small $(\delta + A)$ is also small, hence in the above equation the sines may be replaced by the angles themselves and

$$\mu = \frac{\delta + A}{A}$$

$$\therefore \delta = (\mu - 1)A$$

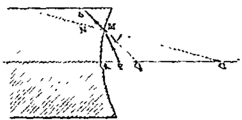


FIG. 106.—Refraction at a Concave Spherical Surface.

EXPERIMENT.—Make a hollow prism out of glass plates with an angle not greater than 10°. Fill it in succession with water and aniline and measure the deviation by the pin method. Taking μ for water as 1·33 find the refractive index of aniline using the last equation.

REFRACTION AT SPHERICAL SURFACES

Image formed by Refraction at a Spherical Surface.—Let AM (Fig. 106) represent a concave spherical surface whose centre of curvature is C and pole A, and let the medium on the left have a refractive index μ relatively to the medium on the right; *e.g.* let the medium on the right be air, that on the left glass. Let P be a small object on the axis, PM any ray meeting the surface; we require to find where the refracted ray MS cuts the axis.

the axis at Q, then Q is the point whose position is to be calculated. CMN is the normal at M, hence $\angle$PMC is the angle of incidence and $\angle SMN = \angle QMC$ the angle of refraction r.

Then
$$\mu = \frac{\sin PMC}{\sin QMC}$$

From $\triangle$ PMC
$$\frac{PM}{PC} = \frac{\sin C}{\sin i}$$

from $\triangle$ QMC
$$\frac{QM}{QC} = \frac{\sin C}{\sin r}$$

dividing the 2nd by the 1st
$$\frac{QM}{QC} \cdot \frac{PC}{PM} = \frac{\sin i}{\sin r} = \mu$$

Limiting ourselves to the case where M is near A, as in the corresponding case for reflexion, we may put QM = QA, PM = PA,

and
$$\mu = \frac{QA}{QC} \cdot \frac{PC}{PA}$$

Put PA = u, QA = v, CA = r and measure all distances from A,

then
$$\mu = \frac{QA}{CA - QA} \cdot \frac{CA - PA}{PA} = \frac{v}{r - v} \cdot \frac{r - u}{u}$$
$$\therefore \mu ur - \mu uv = vr - uv$$

Divide throughout by uvr and rearrange the terms, then

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r}$$

This equation enables us to calculate the distance QA when the other quantities are given. Exactly the same result holds for the rays starting from P, provided they are incident near A, for after refraction all rays diverge from Q and this point is the image of P. The same equation is true for a convex surface if the usual convention is used. Thus in Fig. 107, where the lettering is the same as before,

[illegible]
$$\mu = \frac{\sin LMN}{\sin QMC} = \frac{\sin i}{\sin r}$$

om $\triangle$ PMO $\quad \frac{PM}{PC} = \frac{\sin C}{\sin PMC} = \frac{\sin C}{\sin(\pi - i)} = \frac{\sin C}{\sin i}$

nilarly from $\triangle$ QMC $\quad \frac{QM}{QC} = \frac{\sin C}{\sin r}$

d $\quad \frac{QM}{QC} \cdot \frac{PC}{PM} = \frac{\sin i}{\sin r} = \mu$

$$\therefore \mu = \frac{QA}{QC} \cdot \frac{PC}{PA} = \frac{v}{v + (-r)} \cdot \frac{u + (-r)}{u} = \frac{v}{v - r} \cdot \frac{u - r}{u}$$

in the first case. It should be remembered that u and v are the stances of the object and image respectively from A, and that all

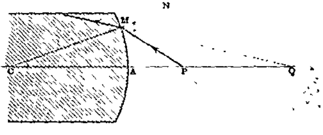


Fig. 107.—Refraction at a Convex Spherical Surface.

distances are to be measured from this point with the usual sign convention. If an eye could be placed in the medium on the left the point P would appear to be at Q. The apparatus used in the following experimental verification of this formula is due to Dr. R. S. Clay.

Experiment.—Pour water into a circular glass crystallising dish 15 cms. or more in diameter and place it on a drawing board. Fasten a pin into a flat block of lead and put it in the water at P (Fig. 108). Look into the water from S and find by the usual pin method the direction of several rays emerging near A. Measure PA = u, and the diameter of the dish; taking μ = 1.33 calculate the distance of the image from A. Rule in the outline of the d[illegible] produce the refracted rays backwards until they meet; m[illegible] QA and compare it with the calculat[illegible]

Example.—The experiment [illegible]
position; the f[illegible]
v = ?

Graphical Construction of Images.—Suppose P in Figs. 106 and 107 is very distant, then the incident rays are parallel with the principal axis, and all the refracted rays meet at a point called the second principal focus; the distance of this point from A is the second focal length. This length f_2 can be found if we put $u = \infty$, and therefore $1/u = 0$, in the above equation; we get $v = f_2 = \frac{\mu r}{\mu - 1}$. Similarly there is a point such that all the rays coming from it are parallel after refraction; this is called the first focal point, and its distance from A is the first focal length f_1. In this case the image is infinitely distant and therefore $\mu/v = \mu/\infty = 0$; hence from the equation we obtain $u = f_1 = -\frac{r}{\mu - 1}$.

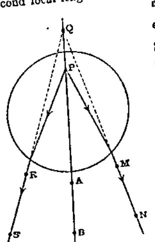


Fig. 108.—Apparatus to verify the Formula for Refraction at a Spherical Surface.

We can make use of these focal points to construct graphically the image of any small object in a manner similar to that employed on p. 153. This is left as an exercise for the student. Refraction at spherical surfaces only becomes of practical importance when the light emerges into the air again, for this two refractions are necessary and we have to deal with a lens.

Lenses

Passage of Light through a Lens.—A lens may be defined as a portion of a transparent, refracting, medium bounded by two surfaces which are most frequently parts of spheres or cylinders. The lenses we shall consider are those bounded by spherical surfaces having a common normal, the plane being regarded as a sphere of infinite radius. Lenses are divided into two classes, those which are ... called convex, those which

est at the middle are concave. Fig 100 shows three of each

The principal axis of a lens is the line joining the centres of ıture of the faces. If one surface is plane the principal axis is endicular to this face and passes through the centre of curvature e other.

XPERIMENT.—Place a glass convex lens on the Hartl disc and allow a er of rays to fall on it parallel with the principal axis; the beam is rendered

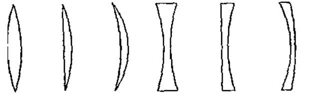


FIG. 100.—Types of Lens.

rergent and provided we deal only with the part near the axis all the rays t at a point F behind the lens (Fig. 110, A) A concave lens causes the rays

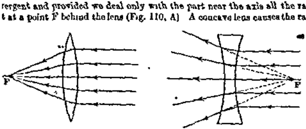


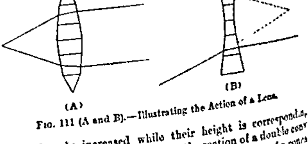
FIG. 110 (A and B).—Passage of Light through a Lens.

diverge, but if the emergent rays are produced backwards they meet at a ınt F in front of the lens (Fig. 110, B).

When a number of rays fall on a lens parallel with the principal is they are made to converge to or diverge from a point; this oint is the principal focus and its distance from the lens is the focal ngth of the lens. With the usual sign convention the focal length f a convex lens is negative, that of a concave lens is positive, all istances being measured from the lens. The focal length is the same

no matter which surface is presented to the incident light. A point on the opposite side of the lens to the principal focus and the same distance away is called the first focal point, the principal focus just defined is then called the second focal point.

This converging or diverging effect of a lens is explained by a reference to Fig. 111, A and B. In the first we have two sets of truncated prisms of different angles arranged symmetrically about an axis with the base of each prism turned towards this line. The prisms furthest away from the axis have the largest angle and they produce the largest deviation of a ray. Since a prism bends rays towards its base such an arrangement will bend all rays towards the axis, or will make a beam more convergent. If the number of



(A) (B)

Fig. 111 (A and B).—Illustrating the Action of a Lens.

prisms is largely increased while their height is correspondingly diminished the figure approximates to the section of a double convex lens. Similarly the second figure shows that the section of a concave lens may be regarded as built up from a large number of truncated prisms with their refracting angles turned towards the axis. The refraction produced by such an arrangement will increase with the refractive index of the material and the angles of the prisms. If a lens is immersed in a liquid whose refractive index is greater than that of the glass the rays are deviated in a direction which is opposite to that obtained in the above experiment; a convex lens then causes a parallel beam to become divergent while a concave lens makes it converge.

Suppose the path of the light to be reversed in Fig 110, A and B, then F becomes the first focal point and it is seen that the incident rays are directed towards or from this point they leave

Optical Centre of a Lens.—Let C, C′ (Fig. 112) be the centres of curvature of the faces of a double convex lens, CC′ the principal axis. Draw from C any radius CP of the right-hand face and from C′ draw a radius C′P′ of the other face parallel to CP. Let Q′P′ be a ray which enters the lens at P′ and emerges at P in the direction PQ. Since the faces at P, P′ are parallel the ray passes through as if the lens were a sheet of glass with parallel sides, the emerging ray is displaced sideways but is parallel to its original direction. Let us

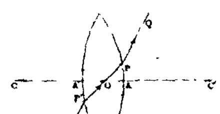


Fig. 112.—Optical Centre of a Lens.

find the position of the point O where the ray in the lens cuts the axis. The △'s OP′C′, OPC are similar,

$$\therefore \frac{OC}{OC'} = \frac{CP}{C'P'} = \frac{CA}{C'A'} = \frac{CA - OC}{C'A' - OC'}$$

the last result following from a well known theorem in algebra.*

Hence $$\frac{CA}{C'A'} = \frac{OA}{OA'}$$

which shows that the point O divides AA′ in the ratio of the radii of the faces. It is therefore a fixed point no matter what pair of parallel radii such as CP, C′P′ are drawn. This point is called the

* If $\frac{a}{b} = \frac{c}{d} = k$ say, then $a = kb$ and $c = kd$, hence $\frac{a - c}{b - d} = \frac{kb - kd}{b - d} = k$. Thus each of the original fractions is equal to $\frac{a - c}{b - d}$.

optical centre of the lens; it is characterised by the fact that all rays which pass through it leave the lens parallel to their original direction. Conversely if the initial and final directions of the rays are parallel it must pass within the lens through the optical centre. When the radii of curvature of the faces have the same sign, as with convexo-concave or concavo-convex lenses, the point at which PP' cuts the axis is virtual, i.e. it lies on PP' produced. In such case the optical centre lies outside the lens and it divides AA' externally in the ratio of the radii, hence, as before, $OA/OA' = CA/C'A'$. If as we shall suppose, the lens is thin the lateral displacement of the rays is small, and it may be said without appreciable error that rays passing through the optical centre continue their course in a straight line.

Graphical Construction of Images.—We are now in a position to find by graphical construction the position of the image formed by a lens. The principles used are similar to those employed for mirrors (p. 153). In the figures here given F is the principal focus as defined on p. 187, for convex lenses it is on the side remote from the source, for concave lenses on the same side as the source, O is the optical centre, and F' is a point on the opposite side of the lens to F such that $OF = OF'$, i.e. F' is the first focal point. The directions of three refracted rays are known for—

(1) Any ray incident parallel with the principal axis passes really or virtually through F after leaving the lens.

(2) Any ray passing through O is undeviated.

(3) Any incident ray which passes through F' is parallel with the axis after refraction by the lens.

To find the image of an object two of these rays are drawn from any point on it and we find where they meet after refraction; this gives one point of the image, others may be found in a similar manner.

To keep the figures clearer the object PQ is supposed to be entirely on one side of the axis. In Fig. 113 *a* it is at a distance from the lens greater than the focal length and the image P'Q', in which all three rays are drawn, is real and inverted. In Fig. 113 the object is nearer the lens than F' and rays (1) and (2) are employed. It is seen that the image is virtual, erect, and magnified. Fig. 1 represents the formation of an image by a concave lens; three rays drawn showing that the image is virtual, erect, and smaller

Conjugate Points.—The distance of the image from the lens can [be] deduced directly from the formula for refraction at a spherical [surf]ace. Let u be the distance of the object from the lens, r_1 and r_2 [the] radii of curvature of the first and second faces respectively, μ

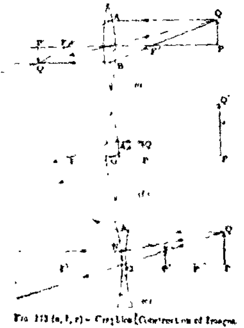


FIG. 113 (a, b, c).—Graphical Construction of Images.

[th]e refractive index of the lens material. For the image formed by [refrac]tion at the first face we have

$$\frac{\mu}{v'} - \frac{1}{u} = \frac{\mu - 1}{r_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

[wher]e v' is its distance from the face. In the lens the rays appear [to] diverge from this image, we may therefore regard it as the [obje]ct when applying the formula to the second face. Let v be the [dist]ance of the final image from the lens, neglecting the thickness of

the glass, and replacing μ by $1/\mu$, since the light is passing from the lens into air, we have at the second face

$$\frac{\frac{1}{\mu}}{v}-\frac{1}{v'}=\frac{\frac{1}{\mu}-1}{r_2}$$

or, multiplying by μ,

$$\frac{1}{v}-\frac{\mu}{v'}=-\frac{\mu-1}{r_2} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Adding equations (1) and (2) together we get

$$\frac{1}{v}-\frac{1}{u}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

The right side is a constant for the lens. When the object very distant the emergent rays pass through the principal fo in this case $v=f$, the focal length, $u=\infty$ and $\frac{1}{u}=0$. Substitu these values we have

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Combining this with the last equation we get finally

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

a formula very similar to that obtained for the image forme reflexion from a spherical mirror. Equations (3) and (4) ar portant and should be remembered. When numerical valu substituted the proper signs must be used; thus for convex f is negative, and for a double convex lens r_1 is negative an positive. Equation (4) can be used to investigate how the im moves when the object changes its position, although generally most useful to draw a figure as in the preceding paragraph. will take the case of a convex lens; from the equation we $v=\frac{uf}{u+f}$, dividing above and below by u and putting the ... of this becomes $v=\frac{f}{\ldots}$. When u is infinitely

$f/u = 0$ and $v = -f$, *i.e.* the image of an infinitely distant object is at the principal focus. For smaller values of u, but such that $u > f$, f/u is < 1, the denominator is positive and < 1, hence $v > f$ and is negative, showing that as the object moves towards the lens from the right the image moves further away to the left. When $u = f$, *i.e.* when the object is at F′ (Fig. 113), $v = -\infty$ or the rays leave the lens parallel with the principal axis. If the object is nearer to the lens than F′ u is less than f and $f/u > 1$, hence the denominator of the fraction is negative and v is positive, meaning that the image is on the same side of the lens as the object and is therefore virtual. For values of u only slightly smaller than f, v is very great and is positive, but as the object approaches the lens the image moves in the same direction and the two coincide at the lens itself. It will be noticed that when the object passes F′ the image moves round from $-\infty$ to $+\infty$.

The reciprocal of the focal length is called the **power of a lens**; if f is given in metres the power is given in diopters. Thus a lens whose focal length is $\frac{1}{2}$ metre has a power 2 diopters.

Linear Magnification.—Expressions for the linear magnification as defined on p. 156, can readily be deduced from Fig. 113 *a*, *b*, *c*. To keep the signs consistent it must be remembered that if PQ (figure, *a*) is taken as positive then P′Q′ must be considered negative since it is drawn in the opposite direction. In order to avoid confusion we shall also find it convenient to put $OF'_0 = f'$ with proper sign and substitute $-f$ for this in the final results, since f and f' are measured in opposite directions. In Fig. 113 *c* we have

Exactly the same formulæ hold for a convex lens; thus in Fig. 113 *a*, from △'s OP′Q′, OPQ

$$\frac{-\text{Image}}{\text{Object}} = \frac{OP'}{OP} = \frac{-v}{u}$$

In △'s FP′Q′, FOA,

$$\frac{-\text{Image}}{\text{Object}} = \frac{FP'}{OF} = \frac{OP' - OF}{OF} = \frac{-v-(-f)}{-f} = \frac{-(f-v)}{f}$$

And from △'s F′PQ, F′OB,

$$\frac{-\text{Image}}{\text{Object}} = \frac{OF'}{PF'} = \frac{OF'}{OP - OF'} = \frac{f}{u-f} = \frac{-f}{u+f}.$$

Changing signs on both sides the last three are the same as before [illegible]

When numerical values are substituted in these expressions [illegible] proper signs must be used, if the result comes out negative it [illegible] that the image is inverted. Conversely if the magnification is [illegible] in (2) or (3) with its proper sign (negative for an inverted [illegible]) the focal length can be calculated if either v or u is known.

If any two of these expressions are equated the ordinary formula is at once obtained. Thus from (1) and (3)

$$\frac{v}{u} = \frac{f}{u+f}$$

$$\therefore uv + vf = uf$$

dividing by uvf and rearranging the required result follows. Th[illegible] perhaps the simplest method of getting the equation as it [illegible] necessary to follow the details of the refraction at each face [illegible] is required is an experimental knowledge of the properties of the [illegible] [illegible] and the optical centre.

Two Lenses in Contact.—For some purposes it is necessary [illegible] [illegible] through two lenses placed in contact; let us calculate the [illegible] [illegible] combination when f_1 and f_2 are the focal lengths [illegible] [illegible] lenses. Let u be the distance of the object [illegible] [illegible] by refraction at the first lens an image is formed [illegible]

garding this image as the object for the second lens, the distance v he final image from the system is given by

$$\frac{1}{v}-\frac{1}{v'}=\frac{1}{f_2}$$

ding the two equations, we have

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f_1}+\frac{1}{f_2}$$

it

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{F}$$

nce

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$$

he focal lengths must be used with their proper signs. The equaon tells us that the power of the combination is the sum of the owers of the components.

Methods of measuring the Focal Lengths of Convex Lenses.—*1st ethod.* The lens is mounted on the optical bench and a real image f the cross-wires is focussed on a screen which can be moved to and ro for this purpose. The distances from the lens of image and object re measured, f is then calculated from the equation $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, with the help of a table of reciprocals. If the same value of f is obtained with different values of u the correctness of the equation is verified.

2nd method. The image of a very distant object is focussed on a screen; the distance between lens and screen is the focal length.

3rd method. If in the first method the wires are placed at the first focal point the rays are parallel after leaving the lens. Suppose they then strike a plane mirror at nearly normal incidence; they will retrace their path and form an image near the cross-wires. Hence mount the lens facing the cross-wires and place behind it on another stand a piece of plane mirror of good quality. Move the lens about until a clear image is obtained beside the cross-wires, the distance of this from the lens is the focal length. An image may be formed by light reflected from the back face of the lens, but this may easily be distinguished from the one sought for since it does not move when the mirror is tilted.

[illegible] method. This is a [illegible] of the [illegible] of the [illegible] [illegible] [illegible] the [illegible] of [illegible] and pin are the [illegible]. [illegible] a [illegible] image is obtained, if the instrument is [illegible] [illegible] a [illegible] image is obtained, [illegible] a [illegible] defined image will be [illegible] [illegible] as any other [illegible] parallel. If the lens is [illegible] on the [illegible] [illegible] parallel [illegible] at a [illegible] of the object [illegible] and back through the [illegible] is clearly [illegible] [illegible] matter. [illegible] the [illegible] and the [illegible] is at [illegible] [illegible] [illegible] focal point of the lens. The advantage of [illegible] and the [illegible] method over the first is that they require shorter distances [illegible] lens and object [illegible] are then less preferable for long focal length [illegible]

[illegible] method. Let P be the cross-wires and S the screen on the optical bench (Fig. 114). When the lens is placed at A an enlarged

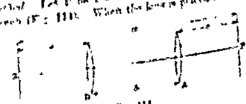


FIG. 114.

image is formed at S, since the path of the light is reversible [illegible] object were placed at S a diminished image would be produced at [illegible] hence if the lens is shifted to B, where PA = BS, a diminished image of the wires is formed on the screen. Let PA = BS = c, PS = a, AB = b, then in either position of the lens we have from the equation

$$\frac{1}{c} - \frac{1}{u} = \frac{1}{f}$$

$$-\frac{1}{b+c} - \frac{1}{c} = -\frac{1}{f}$$

Also

$$a = b + 2c$$

or

$$c = \frac{a-b}{2}$$

Substituting this value in the equation we find

$$f = \frac{a^2 - b^2}{4a}$$

[illegible] from the cross-wires so that [illegible]

images can be found and note how much the lens has to be shifted to change from one to the other; f can then be calculated from the above formula. If the screen is brought nearer to the wires the distance b is diminished until at a certain position only one image, the same size as the object, can be obtained. In this case $b = 0$ and $f = a/4$. It can be shown that the image is now as near to the object as it is possible to get it, hence if this position be found experimentally the focal length is one-quarter of the distance between wires and screen.

6th method. If the magnification and either v or u be measured the focal length can be found from the second or third formulæ on p. 193. As the image is inverted the magnification must be put negative. A slit exactly 1 cm. wide is used as the object, and the image is focussed on a mm. paper scale from which the magnification is read off directly. The following variation gives correct results even for thick lenses. Arrange that the magnification is unity, then, keeping the lens fixed, move the slit and scale until an image is obtained twice, three times, etc., as large as the object. Note the distance through which the scale has been moved from one image to the next; this is the focal length. For, with proper sign,

$$-1 = \frac{f - v_1}{f}$$

or

$$-f = f - v_1$$

Similarly when the magnification is two

$$-2f = f - v_2$$

subtracting one equation from the other

$$f = v_2 - v_1$$

7th method. The following convenient method of determining the power of a lens is due to Prof. S. P. Thompson. The same apparatus is used as in the last method, but the scale is fixed one metre from the lens and the slit is moved until a clear image is obtained. Let this image be m cms. long, then the power of the lens is $(m + 1)$ diopters. For, in the second expression (p. 193) for the magnification, putting $v = -1$ we get, since the image is inverted,

$$-m = \frac{f + 1}{f}$$

whence

$$-\frac{1}{f} = \text{power in diopters} = (m + 1)$$

8th method. All the above methods are inconvenient when the focal length is several metres; in such cases the incident light should be rendered convergent by means of an auxiliary lens of shorter focus. The two lenses may be placed in contact and the formula $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$ employed, or the arrangement shown in Fig. 115 can be used, where C represents the cross-wires, A the auxiliary lens, and B the lens whose focal length is required. With the lens B removed an image is first obtained on a screen at P and AP is measured. B is next placed in position and, the light now being more convergent, the screen has to be moved to Q to receive the image. AB and BQ are measured whence BP = AP − AB can be

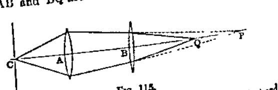


FIG. 115.

found. The image at P is to be regarded as a virtual object and Q is its image formed by the second lens, hence f can be found from the equation

$$-\frac{1}{BQ}-\frac{1}{BP}=\frac{1}{f}$$

Methods of measuring the Focal Lengths of Concave Lenses.—The difficulty in this case is that the image, being virtual, cannot be received on a screen.

1st method. This will be best understood by a reference to Fig. 116. P is a knitting needle which is used as object and Q is its image, M is a small piece of plane mirror with its reflecting surface turned towards a second needle O. An observer on the right sees two images, one of P formed by the lens, the top of this is seen over the edge of M, the other the image of O in the plane mirror; by adjusting the distance OM these can be brought into coincidence at Q, as tested by the parallax method. Then MQ = OM, and LQ = OM − ML; by measuring OM, ML the distance LQ = v can be found, also PL = u can be measured and f obtained from

$$\frac{1}{\ } \quad \frac{1}{\ } \quad \frac{1}{\ }$$

2nd method. The lens is placed in contact with a convex lens of orter focus, the combination forms a convex lens whose focal agth F can be determined, and the required focal length f_2 can be lculated from the equation $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$.

3rd method. The concave lens may replace B of Fig. 115; in iis case the beam is rendered more divergent and the final image is › the right of P. The calculation is the same as before. If the oncave lens is shifted until BP is equal to its focal length the emerent beam is parallel. This parallelism can be tested by either of he methods (3) or (4) of the last paragraph and BP $=f$. Owing, owever, to optical defects it is difficult to get a well-defined mage.

4th method. If a number of convex lenses of known focal lengths

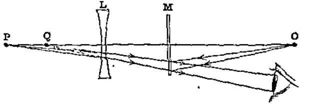


FIG. 116.—Method of finding the Focal Length of a Concave Lens.

are available it may be possible to choose one which just neutralises the concave lens.

EXPERIMENT.—Hold a concave lens close to the eye and look through it at a window frame; if the lens is moved up or down the frame appears to move in the same direction. Repeat with a convex lens, the motions of lens and frame are opposed. Make use of this to find two lenses which just neutralise each other, their focal lengths are equal and opposite in sign. The focal length of the convex lens can be found by the methods given above.

Radii of Curvature of the Faces of a Lens.—Part of the light which falls on a lens is reflected; this may be used to measure the radi[illegible] curvature of the faces. When the face is c[illegible] p. 157 are applicable, if it is [illegible] can be employed, the first of [illegible] generally.

1st

[illegible]

... be found.

2nd method. In this it is arranged that the light which enters the lens meets the second face normally, the rays reflected from this face then retrace their path and form an image at the source, while the transmitted rays emerge into the air without ...

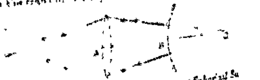


FIG. 114. Method of finding the Radius of a Convex Spherical Surface

refraction. In Fig. 114 P represents the source and C' the point from which the rays diverge after refraction into the lens. Since they meet the second face normally C' is the centre of curvature of this face, it is also the virtual image of P formed by the light which passes through the lens. Its position is therefore given by the usual lens equation $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, and as $v = R_2$, the radius of the back face,

$$\frac{1}{R_2} - \frac{1}{u} = \frac{1}{f}$$

or

$$R_2 = \frac{uf}{u+f}$$

where f is to be used with its proper sign. To carry out the experiment the lens is placed on the optical bench with the convex face whose radius is required turned away from the wires; it is moved about until a clear image is obtained by reflexion from this face. The distance between source and lens is then u of the formula. The focal length must be measured by a separate experiment before ...

can be calculated. The lens may also be floated on a small quantity of mercury to increase the amount of reflected light, a small cardboard pointer is placed above it and this is moved about until it coincides with its own image as tested by parallax. This gives u above.

Refractive Index of a Lens.[1]—When the radii of curvature and the focal length have been measured, the refractive index of the material can be calculated from the equation $\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$.

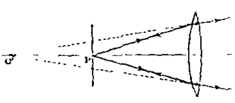


FIG. 118.—Path of Rays reflected from the Back, Convex Face of a Lens.

As exercises on the preceding methods the following experiments are instructive.

EXPERIMENT.—Lay a piece of plane mirror on the floor and place on it a convex lens. Support a cardboard pointer in a stand above the two and move it up and down until it coincides with its real image; the distance from lens to pointer is the focal length f_1. (This is simply a modification of the old method, p. 195.) Run a film of water between the lens and mirror, this forms a plano-concave liquid lens whose upper face has a radius of curvature R_2 the same as the lower face of the glass lens. Measure the focal length F of the combination by the same method, then the focal length f_2 of the liquid lens is given by $\frac{1}{f_2} = \frac{1}{F} - \frac{1}{f_1}$, hence f_2 is known.

Also
$$\frac{1}{f_2} = (\mu - 1)\left(\frac{1}{R_2} - \frac{1}{\infty}\right) = \frac{\mu - 1}{R_2}$$

where μ is the refractive index of the liquid.

If R_2 is measured by any of the preceding methods μ can be found.

EXPERIMENT.—Fill a watch glass with liquid, cover it with a plate of glass to ensure a flat surface and place it on a mirror as in the last experiment. Find,

[1] Barton and Black, "Practical Physics," p. 94.

through a short focus convex lens A (Fig. 117) and falls on the convex surface B. By shifting A or B it can be arranged that the rays meet the surface at nearly normal incidence; when this happens the reflected portion retraces its path and forms an image near the wires. If the rays are produced to the right they evidently meet at C, the centre of the surface. The distance AB is measured, B is removed and a screen is placed at C to receive a well-defined image. The distance AC is measured, whence the radius BC = AC − AB can be found.

2nd method. In this it is arranged that the light which enters the lens meets the second face normally, the rays reflected from this face therefore return along their path and form an image near the source while the transmitted rays emerge into the air without further

FIG. 117.—Method of finding the Radius of a Convex Spherical Surface.

refraction. In Fig. 118 P represents the source and C′ the point from which the rays diverge after refraction into the lens; since they meet the second face normally C′ is the centre of curvature of this face, it is also the virtual image of P formed by the light which passes through the lens. Its position is therefore given by the usual lens equation $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, and as $v=R_2$, the radius of the back face,

$$\frac{1}{R_2}-\frac{1}{u}=\frac{1}{f}$$

or

$$R_2=\frac{uf}{u+f}$$

where f is to be used with its proper sign. To carry out the experiment the lens is placed on the optical bench with the convex face whose radius is required turned away from the wires; it is moved about until a clear image is obtained by reflexion from this face. The distance between source and lens is then u of the formula. The focal length must be measured by a separate experiment before R_2

) Show how the focal length of a convex lens depends on the curvature of ces. (L. '05.)

1 A flat object, whose surface is a sq mm., is placed facing an ordinary nifying glass at a distance u from it; an image of the object is formed at itance mu from the lens. Prove that the size of this image will be such its area is m^2a. Will it make any difference whether the image be real or ual ? (L. '97)

2 Find the size and position of the image formed, by a convex lens of n focal length, of the following object, viz. an arrow 2 ft long, lying along axis of the lens with its middle point 30 in. from the lens. What would ie image if the arrow were turned through 90° 99.)

rangement of a lamp, slit, lens, and scale by the slit formed by the lens and reflected by a n to the scale. Trace in your sketch the course [This is a modification of the lamp and scale ons, see Fig. 89.]

focal length is held 1 in. from the eye by a person sion of 9 in. so as to look at a small object. Where aced ? Illustrate your answer by a figure. (L. '02.)

owing in the case of a convex lens the connexion between ject from one principal focus and the distance of the .. (L. '03) [See next example.]

16. If the distance of an object from the first focal point of a convex lens x and the distance of its image from the second focal point is y, show, without gard to sign, that $xy = f^2$.

17. A convex lens of 10 cms focal length is held in a horizontal position ust above the surface of a liquid filling a tank 20 cms. deep. The image of a oint 30 cms. above the centre of this lens is brought to a focus on the bottom f the tank. Draw a diagram of the path of the rays and calculate the index f refraction for the liquid. (L. '08.)

18. The focal length of a convex lens is 10 in. It is placed in a small tank with parallel sides. Where is the image of a distant object formed if the tank s filled with (1) water, (2) a liquid of refractive index 1·63 ? Take μ for lens and water as 1·53 and 1·33 respectively.

m and the white

19. In method five of finding the focal length of n eye is placed to length of the object and l_1, l_2 the lengths of the the spectrum appears $l = \sqrt{l_1 l_2}$ —V'. Since the rays

20. Use the result of Example 16 to show that ppear to do so, R'—V' real image and object is $4f$ for a convex lens. at its colours are much

21. When a luminous point is placed on the p eal spectrum R—V. and at a distance a from it an image is formed ly side. If a second lens (B) is placed close to acting edge until the minimum the focal length of lens B and state whether id that the spectrum is shorter,

[illegible] the focal length of this glass [illegible]
[illegible] the focal length, [illegible]

$$\frac{\mu_1 - 1}{\mu_2 - 1} = \frac{f_2}{f_1}$$

[illegible] the refractive index μ of the first [illegible]

[illegible]

EXAMPLES ON CHAPTER XVII

1. A small air bubble in a sphere of glass 4 in. in diameter appears [illegible] ... of the bubble and the centre of the sphere are in a line [illegible] ... be 1 in. from the surface. What is its true distance? (μ = [illegible]) (L. [illegible])

2. A small object is enclosed in a sphere of solid glass 7 cms. in radius ... situated 3 cm. from the centre and is viewed from the side to which ... Where will it appear to be if the refractive index of the glass ... 1·5? (L. '91.)

3. A block of transparent jelly of refractive index 1·33 is bounded on ... side by part of a convex surface of a sphere of radius 8 mms. Find the position of the principal focus within the mass of material. (L. '93.)

4. Construct the path of a ray passing through a spherical boundary of a dense medium of given refractive index. Calculate the position of the place to which parallel rays passing nearly perpendicularly through the surface would converge if the refractive index be 1·7 and the radius of curvature 6 ... (L. '92.)

5. A ray of light passing through a prism meets the second face at perpendicular incidence. If i is the angle of incidence on the first face and A the angle of the prism, show that $\mu = \sin i / \sin A$.

6. What is the greatest allowable angle of a prism in order that a ray, incident on the first face at an angle of 60°, may emerge from the second face? ($\mu = 1·64$.)

7. The refracting angle and the minimum deviation for a given prism are each 60°. Find, by means of a diagram showing the course of the minimum ray, the refractive index of the glass. (L. '10.)

8. A lens forms an image one-third the size of an object and 2 ft. distant from itself. What is the focal length of the lens? Where is the object? Consider the case of virtual as well as of a real image. Draw diagrams to illustrate your answer. (L. '90.)

9. A bright point is situated on one wall of a room 9 ft. wide. A lens, 1 ft. focal length and 2 in. in diameter, is placed 3 ft. from the wall ... from the point. What is the width of the circle of [illegible]

fore inclined to the horizontal as in Fig. 120, where A shows the spectrum uced by the first prism and B that produced by the two. The direction e rays, but not their colour, is altered by the second prism.

We have spoken of seven different spectral colours, in reality e are a much larger number; an artist would see many more n seven, while a person whose colour sense is badly developed ild probably see less. It is known n physical optics that the wave-gth of the red rays is nearly double t of the violet. Light of a inite wave-length, and therefore definite colour, is called mono-omatic.

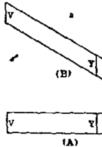


Fig. 120.—Newton's Experiment with Crossed Prisms

Recomposition of White Light.—ice white light is a mixture of ours it ought to be possible to mbine different coloured rays so as produce white light. This can be ne in several ways.—

(1) Fig. 121 represents two prisms, actly alike, with their refracting edges in opposite directions; e dispersion produced by the first is then just cancelled by the cond and the emergent beam is white. The arrangement, in fact, ts like a parallel plate (p. 166) and all the rays emerge parallel to

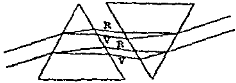


Fig. 121.—Recomposition of White Light by two Prisms.

heir original directions. If a piece of cardboard is held between he prisms to obstruct some of the light the patch is again oloured.

(2) The spectrum produced by a prism may be thrown on to a number of strips of plane mirror from which the light is reflected to a screen. A number of coloured patches are seen which may be made

but its colours are much more brilliant and pure. Hence in the prod
of spectra it is advantageous to place the prism in the position of m
deviation.

The question to be answered now is, Does the prism colo
light during its passage through the glass or does it merely se
colours which are already present? If a piece of blue glass
in the path of the rays between slit and prism only the violet
the spectrum appears, if a piece of red ruby glass is used
only the red end is seen. The prism is therefore unable t
violet light into red, or *vice versâ*, and we conclude that the d

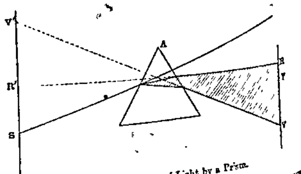


FIG. 119.—Dispersion of Light by a Prism.

lours were originally present in the white light; all the pr
es is to make them visible by separating them from each oth
nce the different coloured rays are deviated by different amou
is clear that the refractive index of the prism varies with th
lour of the light; the violet rays, which are deviated most, a
id to be more refrangible than the red which are deviated lea
is owing to this difference in refrangibility that the rays
parated. That this is the correct explanation of dispersion is s
y Newton's experiment of the crossed prisms.

EXPERIMENT.—In the first experiment above the refracting edge of the
rism is *vertical* and the spectrum is horizontal; hold between the p
e screen a second prism with its refracting edge *horizontal* and
ost. The red rays coming from the first prism fall on the second and
pwards, the violet rays fall on a different part and, owing to their
frangibility, are bent upwards by a larger amount. The final spect

twice spectrum B. By properly choosing the prism angles it ıy be arranged that either (1) the lengths of the spectra, *i.e.* the gular dispersions, are equal, or (2) the deviations of the yellow y are equal. If two prisms of different glass are made to fulfil ndition (1) and are then arranged as in Fig. 121, the dispersion of e first is cancelled by the second, but the deviations of the mean y do not cancel, *i.e.* the beam is deviated without dispersion. Such combination is called an **achromatic prism.** If instead the prisms lfil condition (2), then, when placed as in Fig. 121, they will not eviate the yellow ray, but the beam as a whole will be dispersed. his is the principle of the direct vision spectroscope; the two risms are placed in a tube which carries a slit at one end and a ıagnifying lens at the other, when the slit is direct towards a source

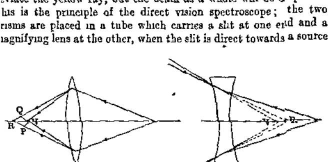


Fig 124.—Dispersion in a Lens.

of light the spectrum can be seen through the lens. This form of spectroscope is more portable than that shown in Fig. 123 but the dispersion it produces is less.

Dispersion in Lenses.—As the refractive index of a material varies with the colour of the light, when white light passes through a lens dispersion will take place and the differently coloured rays will be brought to different foci. The violet rays being the most refrangible their focus will be nearest to the lens. Fig. 124 shows the path of the rays. PQ (figure *A*) shows the position at which a card must be held to obtain the best-defined image. If a screen is held to the right of this point the outer edge of the image will be coloured red, while further to the left it will be violet. These colour effects are usually seen at the edges of the field when a cheap opera glass or telescope is used. For many instruments it is important that they

Spectrum Analysis.—The sp[illegible] … is usually continuous [illegible] from an incandescent solid is usually continuous … under suitable cond[illegible] above, but if a substance is heated under suitable … of bright lines [illegible] spectrum is found to consist of a number of bright lines … characteristic of the substance [illegible] ground. These lines are characteristic of the substance … be used to identify it for purposes of chemical analysis. … in a B[illegible] little strontium chloride is heated to volatilisation in a B[illegible] its spectrum is found to consist of a number of bright red and [illegible] lines. Other methods of heating can also be used.

Angular Dispersion. Dispersive Power.—The angle between [illegible] differently coloured rays after they emerge from the prism is ca[illegible] the angular dispersion for those rays. It varies with the angle [illegible] the prism and the nature of the material. If A is the prism angle and δ_v the deviation produced in a violet ray for which the refractive index is μ_v, then *when A is small*

$$\delta_v = (\mu_v - 1)\mathrm{A} \quad \text{(p. 183)}$$

Similarly for a red ray the deviation δ_r is

$$\delta_r = (\mu_r - 1)\mathrm{A}$$

Hence the angle between the rays, *i.e.* the dispersion, is

$$\theta = \delta_v - \delta_r = (\mu_v - \mu_r)\mathrm{A}$$

If μ is the refractive index for the mean ray (say yellow light) [illegible] quantity $\omega = \dfrac{\mu_v - \mu_r}{\mu - 1}$ is called the **dispersive power** of the material [illegible] Since refractive indices are constants for a given material it [illegible] that the dispersive power depends only on the nature of the [illegible] and not upon its refracting angle.

Achromatism. Direct Vision Spectroscope.—For many p[illegible] it is necessary that rays of light should be deviated with[illegible] persion or be dispersed without deviation. The possibility [illegible] of these can be seen from the following considerations [illegible] number of prisms, *made of different kinds of glass*, be caused [illegible] to form a spectrum under similar conditions, and let the [illegible] of the yellow ray and the length of the spectrum be measured [illegible] case. It will be found that these quantities do not increase [illegible] same ratio when we go from one kind of glass to another [illegible] example, if the deviation produced by prism A for the yel[illegible] twice that produced b[illegible], the length of spectrum A will [illegible]

Spectrum Analysis.—The spectrum formed by the light com
m an incandescent solid is usually continuous like those stud
ove, but if a substance is heated under suitable conditions t
ectrum is found to consist of a number of bright lines on a da
ound. These lines are characteristic of the substance and m
used to identify it for purposes of chemical analysis. Thus
tle strontium chloride is heated to volatilisation in a Bunsen fla
spectrum is found to consist of a number of bright red and g
nes. Other methods of heating can also be used.

Angular Dispersion. Dispersive Power.—The angle between t
ifferently coloured rays after they emerge from the prism is c
ne angular dispersion for those rays. It varies with the angl
he prism and the nature of the material. If A is the prism a
nd δ_v the deviation produced in a violet ray for which the refra
ndex is μ_v, then *when A is small*

$$\delta_v = (\mu_v - 1)A \text{ (p. 183)}$$

Similarly for a red ray the deviation δ_r is

$$\delta_r = (\mu_r - 1)A$$

Hence the angle between the rays, *i.e.* the dispersion, is

$$\theta = \delta_v - \delta_r = (\mu_v - \mu_r)A$$

If μ is the refractive index for the mean ray (say yellow lig
quantity $\omega = \frac{\mu_v - \mu_r}{\mu - 1}$ is called the dispersive power of the m
Since refractive indices are constants for a given material it
that the dispersive power depends only on the nature of th
and not upon its refracting angle.

Achromatism. Direct Vision Spectroscope.—For many p
it is necessary that rays of light should be deviated wit
persion or be dispersed without deviation. The possibility
of these can be seen from the following considerations
number of prisms, *made of different kinds of glass*, be cause
to form a spectrum under similar conditions, and let the
of the yellow ray and the length of the spectrum be measur
case. It will be found that these quantities do not incre
same ratio when we go from one kind of glass to ano
example, if the deviation produced by prism A for the ye
twice that produced by B, the length of spectrum A will n

We have already seen that heat destroys phosphorescence; t property can be used to demonstrate the presence of the infra-r rays.

EXPERIMENT.—Throw an arc-light spectrum on to some Balmain's pa which has previously been made phosphorescent. Where the red and in red rays fall the glow is more vivid for a few seconds, then dies away; rise in temperature they produce causes a more rapid emission of the previou absorbed energy.

If a weak solution of eosine is exposed to light the path of rays can be traced through the whole vessel, but as more eosine added the fluorescence is concentrated at the side where the li enters. This is due to the strong absorption of the active rays by the solution, all the violet light is stopped within a short distance of the surface. If the transmitted light is thrown into a spectrum it will be found that the violet end is missing. The energy of the rays is absorbed and transformed into the fluorescent light.

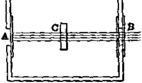


FIG. 126.—Stokes' Method of Detecting Fluorescence.

Stokes' Method of Detecting Fluorescence.—When the fluoresce is weak it may be masked by the dazzling effect of the exciting be Stokes overcame this difficulty by making use of the fact that fluorescent light was more refrangible than that which caused The substance to be examined was placed at C (Fig. 126) in a blackened on the inside and pierced with apertures at A and B. was covered with two sheets of glass, one dark blue, the other gr these stopped all the red, yellow, and green light coming fro source placed just outside. B was covered with a yellow sc which did not transmit blue or violet light. Suppose the subst at C did not fluoresce, the blue light which entered at A was stop by the screen at B and an eye placed near the latter received light, thus the substance was invisible. If, however, there fluorescence the blue light was transformed into green, and could pass through the yellow glass, making the object visible.

Becquerel's Phosphoroscope.—If the phosphorescent light emi by a substance disappears in a small fraction of a second after

nsity of illumination at the point. To render the definition of value it must be made clear in what units Q is to be measured. unit of light is the amount that falls on a screen 1 sq. cm. in held perpendicular to the rays coming from a standard candle ed 1 cm. away. Standard candles are made from sperm, y weigh six to the pound and should burn at the rate of 120 grains hour. Such a standard cannot be regulated easily and is unsfactory for other reasons; it is now replaced for practical purses by a lamp, called the Hefner lamp, which burns amyl acetate. e flame of this is adjusted to a fixed height and the amount of ht it then emits has been carefully compared with that given out a standard candle. When a screen is placed at a distance of m from a standard candle, so that the light falls on it normally, intensity of illumination is unity.

The illuminating power of a source is the ratio of the quantity of

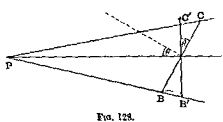


Fig. 128.

it it emits to the quantity emitted in the same time by a standard idle. Let a small screen be held at a cm distance from a standard dle so that the light falls on it normally, its intensity of illumina- is unity; the illumination will become two if we add another dle to the first, to three if three candles are used, and so on. Hence illuminating power of a source is measured by the intensity of nination it produces on a small screen 1 cm. away when the it is incident normally. The unit of illuminating power is the andard candle. An incandescent gas mantle when new has an luminating power equal to about sixty candles.

The illumination of a screen varies with the angle at which the it is incident. Thus let BC (Fig. 128) be a small screen which is ving each second a quantity of light Q from the source P; turn und its mid-point through an angle θ to the position B'C' where

at the same height as the slit which is turned towards them. Two tangular patches of light are seen on the tissue paper; by moving e of the sources it can be arranged that these patches are in contact d equally bright. (It is found easier to judge of this equality ıen they are in contact.) Then A receives light from Q only d B is lighted by P alone, hence

$$\frac{I_Q}{I_P} = \frac{QA^2}{PB^2}$$

Rumford's Photometer.—Another method, which is practically a versal of the one just described, is here used to get the two patches. n front of a white screen is placed an opaque rod C (Fig. 131), P

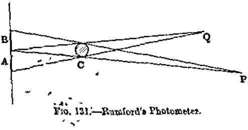


FIG. 131.—Rumford's Photometer.

and Q represent the sources of light. Two shadows of the rod are cast on the screen and, as the figure shows, A receives light from P alone and B from Q alone. One source is moved until the shadows are in contact and are equally dark, then the illumination of A is I_P/PA^2, while that of B is I_Q/QB^2,

hence $$\frac{I_P}{I_Q} = \frac{PA^2}{QB^2}$$

Bunsen's Grease-spot Photometer.—In some form or other this is the one most frequently used.

EXPERIMENT.—Rub a drop of candle grease about the size of a shilling on to a piece of filter paper and when it has set remove most of it with a knife. Hold the paper between the eye and a window; the grease-spot is brighter than the remainder because it transmits more light. If it is viewed from the side from which the light is coming the grease spot is darker than the surrounding paper, for as it transmits more light than its surroundings there is less remaining to be diffusely reflected to the eye.

the source is doubled the quantity of energy received from a given area is reduced to one-quarter if the inverse square law is true, but this decrease is just balanced by using an area four times as large.

Photometry.—An instrument which is used to compare th candle-powers of different sources is called a photometer. It found that the eye is incapable of judging the relative intensities illumination of two surfaces when these are different, but tw observers will agree in a fairly consistent manner in estimating wh two surfaces are equally illuminated. Hence in the comparison illuminating powers it is arranged that two neighbouring patches a screen are illuminated, one by each source, and the distances the sources are adjusted until the patches are equally bright. I I_1 and I_2 be the candle powers of the two sources, R_1 and R_2 th distances from the screen when the two patches are equally bri

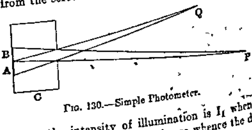


FIG. 130.—Simple Photometer.

From definition the intensity of illumination is I_1 when the source is 1 cm. away from the screen, hence whence the dista R_1 the illumination is I_1/R_1^2, by the inverse square law. Sim the illumination due to the second source at a distance R_2 is I

hence

$$\frac{I_1}{R_1^2} = \frac{I_2}{R_2^2}$$

or

$$\frac{I_1}{I_2} = \frac{R_1^2}{R_2^2}$$

i.e. the illuminating powers are *directly* proportional to the of the distances from the screen when the two patches appear illuminated. A simple photometer can be made as foll piece of brass tube, C (Fig. 130), 5 cms. in diameter and 3 c is closed at one end with thin tissue paper, the other end is ith a brass plate in which has been cut a vertical slit 2 c rd 1 cm. broad . two sources to be compared are arr

)e at the same height as the slit which is turned towards them. Two rectangular patches of light are seen on the tissue paper; by moving one of the sources it can be arranged that these patches are in contact and equally bright. (It is found easier to judge of this equality when they are in contact.) Then A receives light from Q only and B is lighted by P alone, hence

$$\frac{I_Q}{I_P}=\frac{QA^2}{PB^2}$$

Rumford's Photometer.—Another method, which is practically a reversal of the one just described, is here used to get the two patches. In front of a white screen is placed an opaque rod C (Fig. 131), P

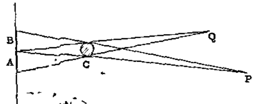


FIG. 131.—Rumford's Photometer.

and Q represent the sources of light. Two shadows of the rod are cast on the screen and, as the figure shows, A receives light from P alone and B from Q alone. One source is moved until the shadows are in contact and are equally dark, then the illumination of A is I_P/PA^2, while that of B is I_Q/QB^2,

hence $$\frac{I_P}{I_Q}=\frac{PA^2}{QB^2}$$

Bunsen's Grease-spot Photometer.—In some form or other this is the one most frequently used.

EXPERIMENT.—Rub a drop of candle grease about the size of a shilling on to a piece of filter paper and when it has set remove most of it with a knife. Hold the paper between the eye and a window; the grease-spot is brighter than the remainder because it transmits more light. If it is viewed from the side from which the light is coming the grease spot is darker than the surrounding paper, for as it transmits more light than its surroundings there is less remaining to be diffusely reflected to the eye.

Such a grease spot arrangement is fixed on an optical b… the sources to be compared are placed on opposite sides of it… the illuminations are the same on each side the light tr… through the grease-spot from right to left is just balanced… coming from left to right and the spot disappears. To… adjustment the lamps are fixed at the ends of the bench… two or more metres long, and the spot is moved to and fro… them. As in the preceding cases, the illuminating pow… directly proportional to the squares of the distances betw… screen and the sources of light. In accurate work the measur… are made in a room with dull black walls, and the spot is pl… a box to screen it from all light except that coming directly… source under test. A pair of mirrors are also fixed at M_1M (F…

FIG. 132.—Grease-spot Photometer.

so that each side of the screen can be seen with the same ey… wise a difference in the eyes might cause error.

EXPERIMENT.—*Use the Bunsen photometer to prove the inverse…* Fix one candle on one side of the spot and a group of four, placed as cl… as possible, on the other. Move the screen until the spot dis… group of four will then be found to be approximately twice as f… screen as the single candle. If the inverse square law is true eac… the group produces at the grease-spot an illumination equal to… produced by the single candle. Hence the illuminations should… intensity when four candles are used at the greater distance, a… experiment shows to be the case. Another, and more satisfacto… is as follows:—Compare the candle-powers of two sources, then… closer together and compare again at various distances apart. As I_1/I… the ratio of the squares of the distances from the grease spot should… this equation is a direct consequence of the inverse square law, h… be proved to hold the inverse square law is true.

EXAMPLES ON CHAPTER XIX

1. How would you determine experimentally the quantity of l… different angles by a … plane glass? (L. '03.)

2. A 10 c p. lamp is placed 1 metre from a surface. At what distances must gas flames of 14 and 16 c p. respectively be placed so as to produce an equal illumination of the surface? (L. '03.)

3. Two lamps A and B are placed 60 cms. and 80 cms. respectively from a Bunsen photometer and it is found that the grease-spot disappears. Find the ratio of the candle powers When a sheet of glass is interposed between lamp B and the photometer it is found that, to produce a balance, this lamp must be displaced 10 cms. Find what percentage of the incident light is reflected by the glass.

4. Light from a 32 c p lamp falls on a silvered mirror and is reflected thence to a grease-spot photometer. The distance from lamp to screen *via* the mirror is 150 cms. If the mirror reflects 90 per cent of the light falling on it where must an 8 c p lamp be placed in order that the grease-spot shall disappear?

5 Two lamps of 8 and 32 c.p. are fixed 120 cms. apart. Where, on the line joining them, must a screen be placed so as to be equally illuminated by each?

CHAPTER XX

THE EYE AND OPTICAL INSTRUMENTS

The Photographic Camera.—One of the simplest applications of the principles explained in the foregoing chapters is the photographic camera. By means of a convex lens a real image of the object to be photographed is focussed on to a glass plate whose surface is coated with certain silver salts. Fig. 113 *a*, p. 191, shows the path of the rays. The blue-violet and ultra-violet rays produce chemical change in the salts which, by treatment with various solutions, are made to produce a permanent record of the image. To obtain good definition an achromatic lens must be used. The linear size of the image is approximately proportional to the focal length of the lens. (See Astronomical Telescope, p. 235.)

The Optical Lantern.—This is an apparatus for throwing on to a screen an enlarged image of an object such as a lantern slide. The optical parts are shown in Fig. 133. AB is the slide, E the achromatic projecting lens, and A'B' the image. Only those rays are drawn which pass through the optical centre of the lens. Owing to the magnification, which is equal to v/u (p. 193), the light which starts from the object is spread over a much larger area in the image; it is therefore necessary that the slide should be strongly illuminated. For this purpose a powerful source of light, such as an arc, is placed at D and the divergent rays are concentrated on to the slide by two large convex lenses, called the condenser, at C. Where a small source has to be used it is an advantage to place a concave mirror at F, then the light travelling to the left, which would otherwise be lost, is reflected back to the condenser and adds to the illumination of the slide. If a piece of apparatus is to be projected it is clear from the figure that its image will be inverted; to obtain the image upright an erecting prism is used. This is a right angled isosceles glass prism st[illegible] its largest face; it is placed between

jecting lens and the screen. Fig. 134 shows how, by reflexion, relative positions of the rays are inverted, resulting in the mation of an upright image.

The Sextant.—This is an instrument which is used to measure the gle subtended at the eye by two distant objects. It is used by lors to determine the sun's altitude, *i.e.* the angle which the line

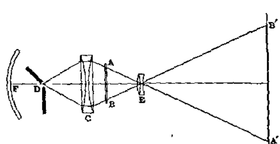


Fig. 133.—Optical Lantern.

going from the observer to the sun makes with the horizontal; this s used in calculating their position when out of sight of land. AB Fig. 135) is a graduated arc of a circle whose centre is C. A movable radius carries a vernier at D and a small vertical mirror at C. A second vertical mirror whose lower half only is silvered is fixed at E, and at T is a telescope. When the mirrors are nearly parallel

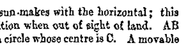


Fig. 134.—Erecting Prism.

suppose the telescope to be directed to some distant object such as a star. A ray FE comes through the unsilvered part of E and enters the telescope, while a parallel ray OC is reflected from C to E and thence also to the telescope. Two images are thus seen and these will coincide when the mirrors are made exactly parallel. The vernier D should then stand at the scale zero at B; if it does not a small correction must be made in subsequent readings. Next let it be required to measure the angle subtended at the observer by two objects situated along CM and EF respectively. That along EF is viewed directly through E and the ray CD ...

[illegible]

Defects of Vision. Spectacles.—The most frequ[illegible]
(1) Short sight, (2) Long sight, (3) Astigmat[illegible].

(1) *Short sight or myopia*.—If the distance betwe[illegible] [illegible] parallel rays are brought to a focus in [illegible] retina and distant objects appear indistinct. Such an [illegible] is myopic or short sighted. As an object moves near[illegible] approaches and finally falls on the retina without acco[illegible] The distance at which this occurs is the maximum distan[illegible] the short sighted person can see distinctly. At clos[illegible] accommodating mechanism comes into play, if this is [illegible] person can see clearly the details of an object which is [illegible] the usual 25 cms. To remedy the defect a lens is req[illegible] shall make parallel rays appear to diverge from a poin[illegible] front of the eye, i.e. a concave lens whose focal lengt[illegible] distant objects can then be focussed distinctly. S[illegible] example, that the range of vision is from 8 cms. to 20 c[illegible] of focal length 20 cms. is required. Let x be the near[illegible] distinct vision through the lens; then a point x cms. [illegible] appear to be 8 cms. distant, or $v = 8$ when $u = x$.

$$\therefore \frac{1}{8} - \frac{1}{x} = \frac{1}{20}$$

and

$$x = 13 \text{ cms.}$$

i.e. the range instead of being from 8 – 20 cms. is now f[illegible] to infinity.

(2) *Long sight or hypermetropia*.—In cases of long sig[illegible] length of the lens is too large and parallel rays are brough[illegible] behind the retina. Consequently the accommodating [illegible] must always be in use and the least distance of distin[illegible] greater than 25 cms. It is only convergent pencils t[illegible] properly focussed with the eye at rest. The glasses to b[illegible] with the purpose for which they are required. Suppose, f[illegible] that the least distance of distinct vision is 40 cms., an[illegible] converging to a point 20 cms. behind the eye can be sharp[illegible] with relaxed accommodation. Then for reading purpos[illegible] of distinct vision must be made normal, while [illegible]

k parallel rays must be clearly focussed with the eye at rest. In first case if an object is 25 cms. distant its image must appear cms. away on the *same side of the lens*, *i.e.* $v = 40$, $u = 25$,

$$\therefore \frac{1}{40} - \frac{1}{25} = \frac{1}{f}$$

$$f = -66 \text{ cms.}$$

a convex lens of 66 cms. focal length is required.

For outdoor work parallel rays must be made to converge to a int 20 cms. behind the eye, *i.e.* a convex lens for which $f = 20$ cms. required. With increasing age the accommodating mechanism comes imperfect and the focal length of the eye lens cannot be tered sufficiently to allow of near objects being sharply focussed. his defect is called *presbyopia*. For example, suppose the nearest stance of distinct vision is 40 cms.; for reading purposes this has be reduced to the normal, and, from the example above, it is seen at a convex lens of 66 cms. focal length is required.

(3) *Astigmatism.*—In some eyes the surfaces of the cornea or the ns do not form parts of spheres, generally a vertical section shows stronger curvature than a horizontal one. In such cases horizontal nd vertical lines are brought to a focus at different distances nd the eye is said to be astigmatic. The necessary correction is btained by the use of lenses which are portions of cylinders.

In some cases the defect may not be the same for both eyes and different lenses must be employed.

Magnifying Power.—Our estimate of the size of an object depends not only on its actual dimensions but also on its distance. Perspective is based on this fact. Thus the metals of a railway appear to approach each other as they recede in the distance, the moon appears to be as large as the sun although it is known to be much smaller, and the height of a distant church spire increases relatively to ourselves as we get nearer to it. In each case we base our estimate on the angle that the body subtends at the eye. In most cases we unconsciously correct our estimate by *making allowance for the distance factor*; when this is impossible or difficult our judgment may be far from the truth; for example, in the case of the sun and moon just mentioned, or in a landsman's estimate of the length of a ship at sea. Similarly when we view an object through a telescope the image may appear greatly magnified, although calculation may

the time light travels from T to M and back again, *i.e.* over a distance $2l$.

Hence
$$V = \frac{2l}{t} = \frac{2l \times 720 \times 2}{T}$$

In one experiment Fizeau found that the wheel made 12·6 revolutions per second when the first eclipse took place and $l = 8633$ metres, hence $T = 1/12{\cdot}6$. Substituting these values $V = 313{,}000{,}000$ metres/sec.

This is rather larger than the usually accepted value, which is about 300,000,000 metres/sec.

Fig. 148 shows the apparatus used. Light from the source Q

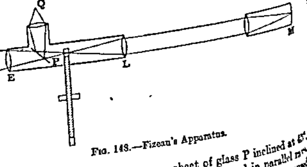


Fig. 148.—Fizeau's Apparatus.

passed through a lens and fell on a sheet of glass P inclined at 45°, from this it was reflected to a lens L and emerged in parallel rays. About 8000 m. away these rays fell on a second lens which caused them to converge on to the mirror M. From this they retraced their path to the plate P. Some of the returning light passed directly through the lens E and thence to the eye, hence the eclipses could be observed. It will be seen that EL really forms a telescope with the rim of the wheel T at the common focus of the two lenses.

Foucault's Method.—This method can be worked in a room of moderate size. Its principle is shown in Fig. 149. S is a small illuminated narrow slit, P a plane mirror which can rotate rapidly round a vertical axis at O, M a concave mirror whose centre of curvature is at O. Suppose the mirror is at rest in the position shown, light from S is reflected to M, and, meeting this mirror

ormally, retraces its path to S. Suppose next that the mirror OP rotating. While the light is going from O to M and back again e mirror turns through a small angle θ into the position OP'. The

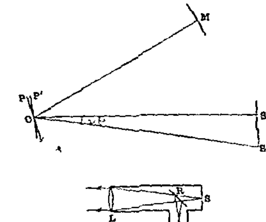


FIG. 149.—Foucault's Method of measuring the Velocity of Light.

ys are now reflected along OS' and the image of the slit at S'.

Also $$\angle SOS' = 2\theta \text{ (p. 141)}$$

$$= \frac{SS'}{OS}$$

r $$\theta = \frac{SS'}{2OS}$$

Hence by measuring these two distances θ can be found. Let t be the time taken by light to travel from O ... again, T the period of revolution of the mi...

Then $V = 2OM/t$, ...

takes to ...

called into play when the body is deflected—in most cases this arises from the elasticity, (2) the body must be capable of storing potential energy, (3) capable of possessing kinetic energy. A pendulum provides another illustration of a vibrating body; here the force of restitution arises from the weight of the bob, and the potential energy is the weight of the bob multiplied by the vertical height through which it has been raised. If the end of the metre stick above is deflected by hanging weights to it a few simple measurements will show that the deflexions are proportional to the weights applied, *provided the deflexions are small.* But in equilibrium the hanging weight is balanced by the force of restitution; it follows that this force is proportional to the displacement of the end of the stick, and is in a direction tending to restore it to its undisturbed position. For small displacements this is true of all vibrating bodies, the force of restitution is proportional to the displacement. Now the acceleration of a body is proportional to the force which acts on it, hence when it is performing *small* vibrations its acceleration is directed towards its mean position and is proportional to the force of restitution, *i.e.* to its displacement. When a particle moves along a straight line so that its acceleration is always directed towards a point in this line and is proportional to the displacement therefrom the particle is said to move harmonically or with a simple harmonic motion. This term is usually abbreviated to the letters S.H.M. In most cases of sounding bodies the displacements are small and the motion is harmonic, it will therefore be convenient if we study first some of the features of simple harmonic motion.

... a circle of radius a ...

left as negative. Let us take as the zero of time the instant [M] is passing through its mean position O in the positive direc[tion; a]t this moment the radius OP is crossing OY. After a time t [OP ma]kes an angle θ with OY; this angle is called the phase of the [moti]on. P is called the generating point and the circle APB the [gener]ating circle. In a time T the point P moves round the circle, [th]rough a distance $2\pi a$, hence

$$2\pi a = vT \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

[Le]t ω be the angle through which the radius OP revolves in 1 sec., [thi]s called the angular velocity of P. In a time T the radius

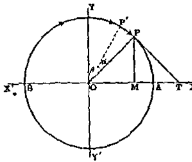


FIG. 150 —Simple Harmonic Motion.

[turn]s through an angle ωT, but this is the angle described in one [com]plete revolution,

[T]herefore $\qquad \omega T = 2\pi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$

[Fro]m (1) and (2) $\qquad v = \frac{2\pi}{T} \cdot a = \omega a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$

[Th]is equation gives an important relation between the linear velocity [of] P and its angular velocity round O.

It is shown in books on mechanics that the acceleration of [P] is directed towards O and is equal to v^2/a. To find the [ac]celeration of M we have to resolve the acceleration of P [al]ong OA. Hence the acceleration f of M is directed towards O;

[illegible]

If the acceleration of P is represented by OP the component [illegible] is represented by OM

$$\therefore f = \frac{a^2}{a} \cdot \frac{OM}{OP}$$

[illegible] from (1)

$$f = \frac{a^2 t^2}{a} \cdot \frac{OM}{OP} = \frac{a^2 t^2}{a} \cdot \frac{x}{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$f = n^2 x$$

Hence the acceleration of M is towards O and is proportional to the displacement, i.e. its motion is a S.H.M.

From equation (2) it is seen that the period $T = 2\pi/n$ [illegible] whenever the acceleration of a point moving along a line is proportional to [illegible] (displacement × constant) we conclude that the motion is harmonic and from (2) its period is 2π divided by the square root of the constant. Also the angle θ is described in the time t.

hence $\theta = \omega t$

and the displacement $x = OP \cdot \sin\theta$

or $x = a \sin \omega t \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$

Whenever the displacement is given by an equation like (3) it may be concluded that the motion is a S.H.M., and the period obtained by dividing 2π by the coefficient of t. If a perpendicular PN is drawn to OY it can be shown in a similar manner that N moves with a simple harmonic motion along YY′. The displacement is given by $y = a \cos \omega t$, this equation therefore represents a S.H.M. The maximum displacement of M from its mean position is called the amplitude of the vibration; it is the radius a in the figure. If a second point P′ move round the circle with the same velocity as P; the angle $\alpha = POP'$ is called the phase difference of P and P′. The displacement of M′, the foot of the perpendicular from P′ on ox_1 is

$$x' = a \sin P'OY = a \sin(\theta - \alpha)$$

$$= a \sin(\omega t - \alpha)$$

nce the velocity of M is $v \cos\theta$, or, in terms of the displace- and angular velocity,

$$\text{vel. of M} = a\omega \cdot \frac{PM}{OP} = \omega \cdot PM$$

$$PM^2 = a^2 - x^2$$

$$\therefore \text{vel. of M} = \omega\sqrt{a^2 - x^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

uppose now that M is a material particle of mass m; since its eration is towards O there must be a force F acting upon it ng to bring it back to this point, and F = mass × acceleration $\omega^2 x$. The potential energy of the particle in any position is work done against this force of restitution. At A it is momen- y at rest and all its energy is potential; let us calculate the gy at this instant. The force is proportional to the displacement, efore the average force is that which acts when the displacement 2.

But $$F = m\omega^2 x$$

$$\therefore \text{average force} = m\omega^2 \frac{a}{2}$$

the displacement at A is a,

$\therefore$ work done in displacing M from O to A

$$= \text{average force} \times \text{total displacement}$$
$$= \tfrac{1}{2}m\omega^2 a \times a$$
$$= \tfrac{1}{2}m\omega^2 a^2$$

This is the potential energy at A. At O all the energy is kineti d must have the value just given. This follows also directly, fo

$$\text{kinetic energy at O} = \tfrac{1}{2}m \times \text{vel.}^2$$

d the velocity at O is equal to the velocity of P, *i.e.* is ωa. We g e same result by putting $x = o$ in Equation (6), then

$$(\text{velocity})^2 = \omega^2 a^2$$

$$\therefore \text{kinetic energy at O} = \tfrac{1}{2}m\omega^2 a^2$$

his result shows that the energy of the vibrating particle is pr rtional to the square of the amplitude. It should be noticed th nce $T = 2\pi/\omega$ the period is independent of the amplitude.

4. Graphical Representation of a S.H.M. ...

a line OX (Fig. 151) and mark ... In the figure these are ...
intervals of time. In the figure these are ... displacements ...
he period. Let positive and negative displacements ... this line.
ed by ordinates drawn above and below this line. after T/4 ...
ginning of time the displacement is zero; after T/4 ...
g. 150) and the displacement is a. Hence at the poi...
4, (Fig. 151), an ordinate of length a is drawn. After ha...
e displacement is again zero, and so on. The thin c...
gure is the result of joining all the points so found. ...
isplacement $x = a \sin \theta$ it follows that the displaceme...
the figure is a sine curve. When the amplitude and th...
known the displacement can be calculated from a table...
as many points on the curve as we choose can be found.
at any instant can be similarly represented by a velocity...
velocity, from the last paragraph, is $v \cos \theta$, and the cur...
is a cosine curve. Thus, of course, is the same shape...
sines, but its maximum ordinate is v when $\theta = 0$ ...
The two curves are displaced relatively to each other...
of time by a distance T/4. These results can also ...
sidering the velocity in Fig. 150. When the displ...
the velocity is v, when the displacement is a the vel...
so on. The dotted curve in Fig. 151 is the curve ...
should be remembered that the curves do not ...
displacements or velocities are parallel to OY but ...
magnitudes are proportional to the corresponding ...

Simple Pendulum as an Illustration of S.H.M.— ...
a motion which is nearly simple harmonic the si...
be taken. In Fig. 152 O is the point of supp...
mass m, and OP $= l$ is the length of the string ...
line PR to represent the weight mg of the bob. ...
into components along and perpendicular to ...
perpendicular to OP and RS perpendicular to ...
represent the two components. The compon...
the tension in the string, while PS represents ...
to bring the pendulum back to its mean posi...

PS = PR ...

Hence restoring force ...

e their maximum displacements at the beginning of the time o
rvation. It is clear that the resultant is no longer simpl
nonic, and that it alternates between relatively very large and
small amplitudes. At the beginning, when the two have th
e phase, the resultant is large, but when the quicker has made
ibrations the slower has made only 2 and the phases are exactly
site; the resultant at this instant is therefore small. When
quicker has made 5 vibrations the slower has made 4, the phases
gain equal and the resultant is large. It is evident that the

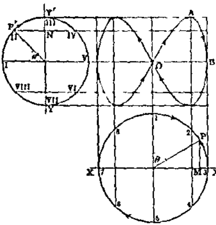


FIG. 155.—Composition of Two Rectangular S.H.M.'s.

num displacement occurs every fourth vibration of the slower
This result will be useful later.

sultant of Two S.H.M.'s at Right Angles to each other.—The
d of finding the resultant in each cases will be best understood
n example. Draw two generating circles P and P' as in Fig.
th their radii proportional to the amplitudes of the motions.
suppose the frequency of P' is twice that of P. Draw two
ers XX', YY', at right angles to each other and drop the
diculars PM, P'N. The points M and N then pe
s. Divide the circumferences into a number of eq
r them as in the figure, and draw lines through th

the book itself is moving parallel to this line with a S.H.M. of the same period but of different amplitude and phase. The actual displacement of M is the resultant of the two motions. This displacement can be obtained graphically. Let the thin curve in Fig. 153 represent the displacement of M at different times if the book were at rest, and let the dotted curve represent the displacement of the book alone at corresponding instants. In drawing the figure it has been supposed that the phase of the second motion is exactly a quarter of a period in advance of the first. Then at the instant represented by A if M alone were moving the displacement would be AB while if the book were moving and M were at rest the displacement would be AC. Hence an ordinate AD = AB + AC represents the actual displacement, and similarly at any other instant the displacement of M is the algebraic sum of the ordinates

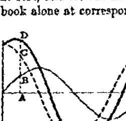


FIG. 153.—Composition of Two S H M.'s in the same Straight Line.

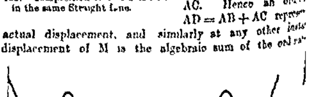


FIG. 154.—Resultant of two S H M.'s in the same Straight Line whose Periods are as 5 : 4.

of the two curves. The thick curve represents the resultant displacement curve obtained in this manner. It is seen that the resultant of two S H M.'s of equal periods in the same straight line is itself a simple harmonic motion. The same method may be followed if the periods are unequal. Fig. 154 represents the resultant of two S H M.'s whose periods are as 5 : 4, the dotted curve represents [illegible]

e their maximum displacements at the beginning of the time of ervation. It is clear that the resultant is no longer simple nonic, and that it alternates between relatively very large and small amplitudes. At the beginning, when the two have the e phase, the resultant is large, but when the quicker has made ibrations the slower has made only 2 and the phases are exactly site; the resultant at this instant is therefore small. When quicker has made 5 vibrations the slower has made 4, the phases again equal and the resultant is large. It is evident that the

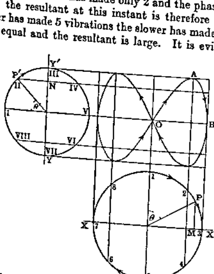


FIG. 155.—Composition of Two Rectangular S.H.M.'s.

num displacement occurs every fourth vibration of the slower

This result will be useful later.

sultant of Two S.H.M.'s at Right Angles to each other.—The d of finding the resultant in such cases will be best understood n example. Draw two generating circles P and P′ as in Fig. ith their radii proportional to the amplitudes of the motions. suppose the frequency of P′ is twice that of P. Draw two ers XX′, YY′, at right angles to each other and drop the diculars PM, P′N. The points M and N then perform 's. Divide the circumferences into a number of equal parts, them as in the figure, and draw lines through these points

orm a network on the right. The points are numbered so that 1
respond to the instants of zero phase, *i.e.* to the instants wl
and M are moving through their mean positions in the posit
ections. If a point is subject to the two S.H.M.'s its position
y instant is determined by the intersection of the two lines dr
rough the corresponding positions of the generating points. T
ppose P is at 1 when P′ is at I, and the point in question is a
hen P′ has reached III P has arrived at 2 and the point is at A
he intersection of the lines through III and 2. When P′ reach
′ is at 3 and the point is at B, and so on. The curve shows the c
lete path of the point. The curves in Fig. 156 have been obta

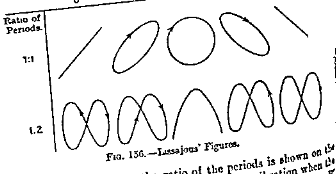


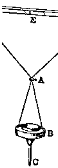
Fig. 156.—Lissajous' Figures.

by a similar construction; the ratio of the periods is shown on the left, while above is shown the phase of the faster vibration when the slower is at zero phase, *e.g.* a phase difference $3\pi/4$ means that P′ has revolved through this angle and is therefore at IV when P is at I. These figures can be obtained experimentally by various devices of which only two need be described.

Blackburn's Pendulum.[1]—A thin string about 7 ft. long has its ends tied to a rod E which is fixed horizontally (Fig. 157). A ... end of the loop a heavy lead ring B is fastened; this carries a ... funnel whose exit tube is fairly narrow. The string can be ... ut up as shown in the figure by a clip at A. The whole arrange... nt forms a pendulum whose length is EB for vibrations perpe... ular to the plane of the figure, but for vibrations in the plane of ...

[1] Barton and Black "Practical Physics," p. 17. Other figures are there gi...

gure it behaves like a pendulum of length AB. Hence if the is pulled outwards in a slanting direction and then released the motions are combined; their relative periods can be adjusted oving the clip A. A record of the motion can be obtained by ng fine, dry, sand in the funnel and allowing it to run out on a of paper placed immediately below. The purpose of the lead s to keep the height of the centre of gravity of the bob constant e sand escapes, otherwise the periods would vary.



157.—Blackburn's Pendulum.

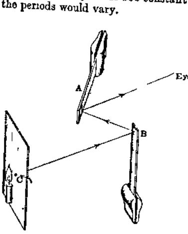


FIG. 158.—Apparatus to produce Lissajous' Figures

sajous' Figures.—Two thin metal strips are supported as in Fig. 158, so that one, A, can oscillate in a horizontal and ier, B, in a vertical plane. Each strip carries at its free end piece of plane mirror. Light from a small hole C in a card-screen falls on the mirror B, whence it is reflected to A and to the eye. If the strip B alone vibrates the spot of light is out into a vertical line, while if A only is in motion the line is ital. When both are oscillating together the two S.H.M.'s abined. The figures can be projected on a screen if a convex placed between C and B. When produced optically in some anner the curves are usually called Lissajous' figures. y have an important application in the comparison of the

...ring by an electric current the sound can be heard directly, but if a [illegible] ... fits in to A note the transmission of the vibrations through the wires ... of the jar to the table top.

In order that the medium may transmit vibrations it m... ...ssess elasticity and be capable of storing potential and ki... ...ergy (p. 271).

(3) *Sound waves should be capable of reflexion and refraction.* ...xperiments given later show that this also is true.

(4) *The waves should be diffracted, i.e. they should bend ro...* ...stacles just like ripples on water bend round a stone and co...

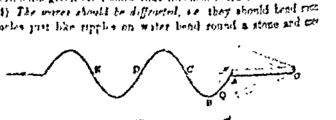


Fig. 159.—Waves along a Wire.

...ain behind it. It is a common experience that we can hea... ...son's voice round the corner of a house.

(5) *The waves should interfere.*—If the medium at a certain p... ...cted upon by two sets of waves, so that the displacements ... each are equal and opposite at every instant, then the medi... ...uld be at rest. This also is established by experiment.

Transverse Waves.—Let OA (Fig. 159) represent a rod with ... fixed at O and the other fastened to a long, *loosely* stretch... If the rod is made to vibrate a number of loops travel alo... as shown in the figure. The particles vibrate up an... ...ile the disturbance travels to the left. In the figure th... ... has reached its maximum displacement downwards and ... point of being pulled up again by the tension. Each part ... string undergoes in succession the same motion as the end o... ..., but the instant at which this occurs depends on its distance...

noticed is that although the particles only oscillate about
position their energy is carried along by the wave, matter
move to the left but energy does. The distance between su
particles in the same phase of vibration is called the wave
In the figure the wave length is CE, for each of these points
mean position and is about to move downwards. D is al
mean position but is on the point of moving upwards, CD
fore half a wave-length. During one complete vibration of
the wave travels a distance CE or AD, the wave-length is t
the distance the disturbance travels in the periodic time
velocity of the wave is the distance it travels in one second.
be the velocity, n the frequency of the vibration, and λ th
length. Then in 1 sec. the rod makes n vibrations and t
travels a distance $n\lambda$, hence $V = n\lambda$. Also $T = 1/n$,
$VT = \lambda$. It can be shown that $V = \sqrt{\frac{F}{m}}$ cms./sec. if
tension of the string in dynes, and m the mass in gms. of 1 cm

Waves of this type are called **transverse**, because the
ments of the individual particles are transverse to the dir
which the wave advances. If the displacement of A
harmonic and is represented by $a \sin \omega t$ or $a \sin \frac{2\pi}{T} \cdot t$,
displacement of a point Q at the same instant is
$a \sin\left(\frac{2\pi}{T} \cdot t - a\right)$, where a is the phase difference between
But the phase difference going from A to D increases by
one is a period behind the other, hence the phase
$a = \frac{x}{AD} \cdot 2\pi = \frac{x}{\lambda} \cdot 2\pi$ behind A. Thus the displacement of
instant is given by $a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$ By putting in ap
values of x and t this expression gives the displacemen
particle of the string; it therefore represents a simple
wave of wave-length λ and period T advancing in the di
which x increases. The velocity of all the particles at an
can be represented by a velocity curve. In Fig. 159 A
upwards with its maximum velocity while B is momen
rest. The lower curve in the figure is the curve of veloci

Longitudinal Waves.—Let us suppose now the rod OA is

kind of large ear-trumpet whose lower end, sunk in the water, was closed by a flexible membrane while the upper end was applied to the ear. The time taken by the sound to travel over a known distance in the water could thus be found. The velocity was found to be 1435 metres/sec. Methods of measuring the velocity in solids are given later.

EXAMPLES ON CHAPTER XXIII

1. Explain why the rise of temperature due to compression and the fall of temperature due to rarefaction in a sound wave both tend to raise the vel... of propagation of the wave. (L. '84.)

2. How does the velocity of propagation of sound through a gas vary with the specific gravity and temperature of the gas? The specific gravities of oxygen and nitrogen gases are as 16 : 14. At what temperature will the velocity of propagation of sound through oxygen be the same as that through nitrogen at 15° C.? (L. '93.)

3. Upon what properties of a solid does the speed of sound in the solid depend? Are all kinds of waves in a solid propagated with the same speed? Why does sound travel faster in steel than in air? (L. '97.)

4. Show that the expression $y = a \sin \frac{2\pi}{\lambda}(x - vt)$ represents a train of waves of amplitude a and wave-length λ moving along the x axis with velocity v. Draw curves showing the variation of the displacement y (1) with the time at the point $x = \frac{\lambda}{2}$ and (2) with x at a time $t = \lambda/v$. (L. '99.)

5. A litre of hydrogen at N.T.P. weighs 0.0896 gm. Find the velocity of sound in hydrogen at a temperature 16° when the pressure is 750 mm., the ratio of the specific heats being 1.4, density of mercury 13.6, $g = 980$.